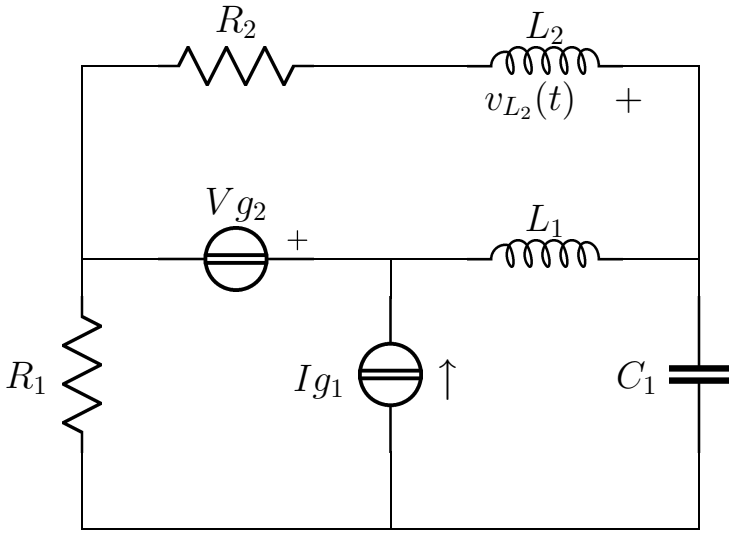
Esercizio ggcesame$_2$016 − 01 − 29$_S$erie1$_M$aglie

Risolvere il circuito in figura



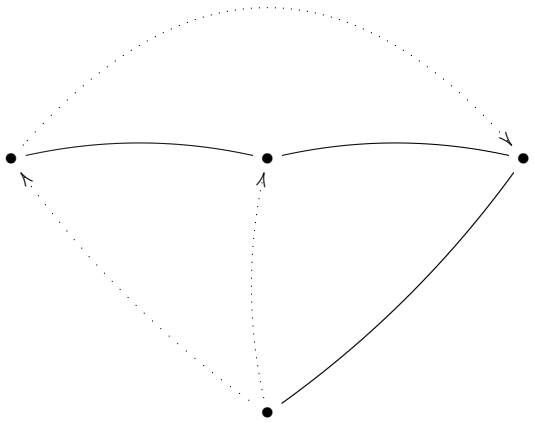
$R_1 = 1$
$\mathbf{I_{g_1}} = 3 - j$
$C_1 = 1$
$\mathbf{V_{g_2}} = -1$
$L_1 = 1$
$R_2 = 1$
$L_2 = 1$
$\omega = 1$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_2 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (R_1 + \frac{1}{j\omega C_1} + j\omega L_1)\mathbf{I_1} & +(\frac{1}{j\omega C_1} + j\omega L_1)\mathbf{I_2} & -j\omega L_1\mathbf{I_3} & = & \mathbf{V_{g_2}} \\ (\frac{1}{j\omega C_1} + j\omega L_1)\mathbf{I_1} & +(\frac{1}{j\omega C_1} + j\omega L_1)\mathbf{I_2} & -j\omega L_1\mathbf{I_3} & = & \mathbf{V_{x_1}} \\ -j\omega L_1\mathbf{I_1} & -j\omega L_1\mathbf{I_2} & +(j\omega L_1 + Z_a)\mathbf{I_3} & = & -\mathbf{V_{g_2}} \\ & \mathbf{I_2} & & = & \mathbf{I_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} \mathbf{I_1} & & -j\mathbf{I_3} & = & -1 \\ & & -j\mathbf{I_3} & = & \mathbf{V_{x_1}} \\ -j\mathbf{I_1} & -j\mathbf{I_2} & +(1+2j)\mathbf{I_3} & = & 1 \\ & \mathbf{I_2} & & = & 3-j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & -1+j \\ \mathbf{I_2} & = & 3-j \\ \mathbf{I_3} & = & 1 \\ \mathbf{V_{x_1}} & = & -j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = \mathbf{V_{x_1}} = -j & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{1}{2} - \frac{3}{2}j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_1} - \mathbf{I_3} = -2 + j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 1 + \frac{1}{2}j \end{array}$$

$P_{c_{tot}} = \frac{3}{2} - j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_1} = -1 + j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \mathbf{I_3} = 1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = (-\mathbf{I_1} - \mathbf{I_2})\frac{1}{j\omega C_1} = 2j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -2 \\ \mathbf{I_{L_1}} = \mathbf{I_1} + \mathbf{I_2} - \mathbf{I_3} = 1 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{L_2}} = -\mathbf{I_3} = -1 & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -1 = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{V_{L_2}} = -\mathbf{I_3} j\omega L_2 = -j$$

$$v_{L_2}(t) = \cos\left(t - \frac{\pi}{2}\right)$$

Soluzioni:

$V_{R_1} = 1 - j;$	$I_{R_1} = -1 + j;$	$Pa_{R_1} = 1$
$V_{g_1} = -j;$	$I_{g_1} = 3 - j;$	$Pc_{I_{g1}} = \frac{1}{2} - \frac{3}{2}j$
$V_{C_1} = -2j;$	$I_{C_1} = -2;$	$Q_{C_1} = -2$
$V_{g_2} = -1;$	$I_{g_2} = -2 + j;$	$Pc_{V_{g2}} = 1 + \frac{1}{2}j$
$V_{L_1} = -j;$	$I_{L_1} = 1;$	$Q_{L_1} = \frac{1}{2}$
$V_{R_2} + V_{L_2} = -1 - j;$	$I_{R_2} = I_{L_2} = 1;$	$Pa_{R_2} = \frac{1}{2}$
$Q_{L_2} = \frac{1}{2}$		