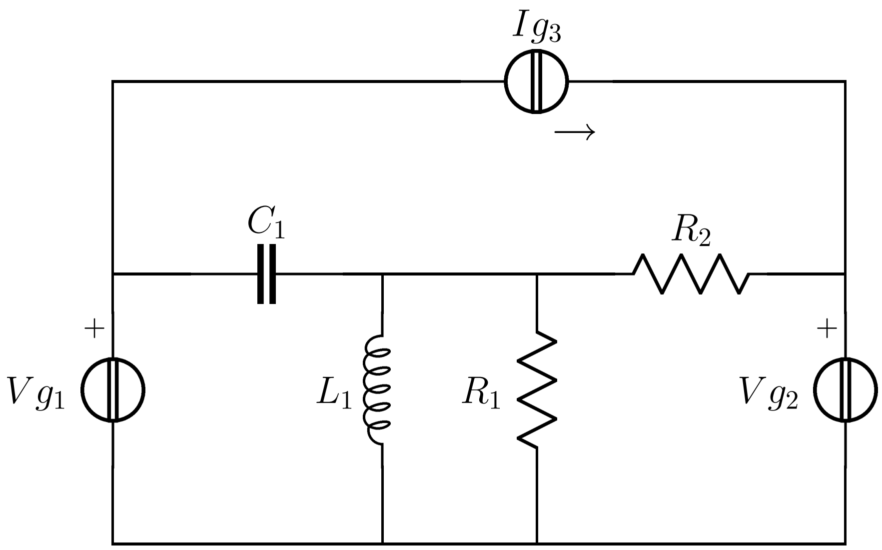
Esercizio ggcesame$_2 015 - 02 - 10_B 1_M aglie$

Risolvere il circuito in figura



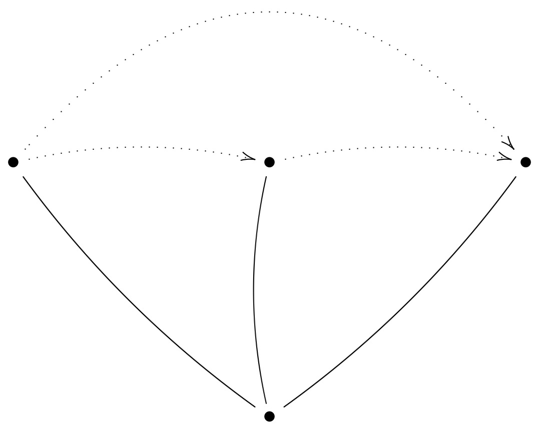
$\mathbf{V_{g_1}} = 2$
$R_1 = 1$
$L_1 = \frac{1}{2}$
$\mathbf{V_{g_2}} = 3 - 4j$
$C_1 = \frac{1}{2}$
$R_2 = 2$
$\mathbf{I_{g_3}} = 1 - j$
$\omega = 2$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_1} = 1 - j$
$Z_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (Z_a + \frac{1}{j\omega C_1})\mathbf{I_1} & -Z_a\mathbf{I_2} & = & \mathbf{V_{g_1}} \\ -Z_a\mathbf{I_1} & +(Z_a+R_2)\mathbf{I_2} & = & -\mathbf{V_{g_2}} \\ & 0 & = & \mathbf{V_{g_1}} - \mathbf{V_{g_2}} + \mathbf{V_{x_3}} \\ & \mathbf{I_3} & = & \mathbf{I_{g_3}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2}-\frac{1}{2}j)\mathbf{I_1} & +(-\frac{1}{2}-\frac{1}{2}j)\mathbf{I_2} & = & 2 \\ (-\frac{1}{2}-\frac{1}{2}j)\mathbf{I_1} & +(\frac{5}{2}+\frac{1}{2}j)\mathbf{I_2} & = & -3+4j \\ & 0 & = & -1+4j+\mathbf{V_{x_3}} \\ & \mathbf{I_3} & = & 1-j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & j \\ \mathbf{I_2} & = & -1+2j \\ \mathbf{I_3} & = & 1-j \\ \mathbf{V_{x_3}} & = & 1-4j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_1} + \mathbf{I_3} = 1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = 1 \\ \mathbf{I_{V_{g2}}} = -\mathbf{I_2} - \mathbf{I_3} = -j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 2 + \frac{3}{2}j \\ \mathbf{V_{I_{g3}}} = \mathbf{V_{x_g2}} = 1-4j & P_{c_{I_{g3}}} = \frac{1}{2}\mathbf{V_{I_{g3}}}\mathbf{I^*_{g3}} = \frac{5}{2} - \frac{3}{2}j \end{array}$$

$P_{c_{tot}} = \frac{11}{2}$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{(-\mathbf{I_1}+\mathbf{I_2})Z_a}{R_1} = -1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \mathbf{I_2} = -1+2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 5 \end{array}$$

$P_{a_{tot}} = \frac{11}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{I_1}\frac{1}{j\omega C_1} = 1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{1}{2} \\ \mathbf{I_{L_1}} = \frac{(-\mathbf{I_1}+\mathbf{I_2})Z_a}{j\omega L_1} = j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = 0 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll}
V_{g_1} = 2; & I_{g_1} = 1; & Pc_{V_{g1}} = 1 \\
V_{R_1} = V_{L_1} = 1; & I_{R_1} + I_{L_1} = -1 + j; & Pa_{R_1} = \frac{1}{2} \\
Q_{L_1} = \frac{1}{2} & & \\
V_{g_2} = 3 - 4j; & I_{g_2} = -j; & Pc_{V_{g2}} = 2 + \frac{3}{2}j \\
V_{C_1} = -1; & I_{C_1} = j; & Q_{C_1} = -\frac{1}{2} \\
V_{R_2} = 2 - 4j; & I_{R_2} = -1 + 2j; & Pa_{R_2} = 5 \\
V_{g_3} = 1 - 4j; & I_{g_3} = 1 - j; & Pc_{I_{g3}} = \frac{5}{2} - \frac{3}{2}j
\end{array}$$