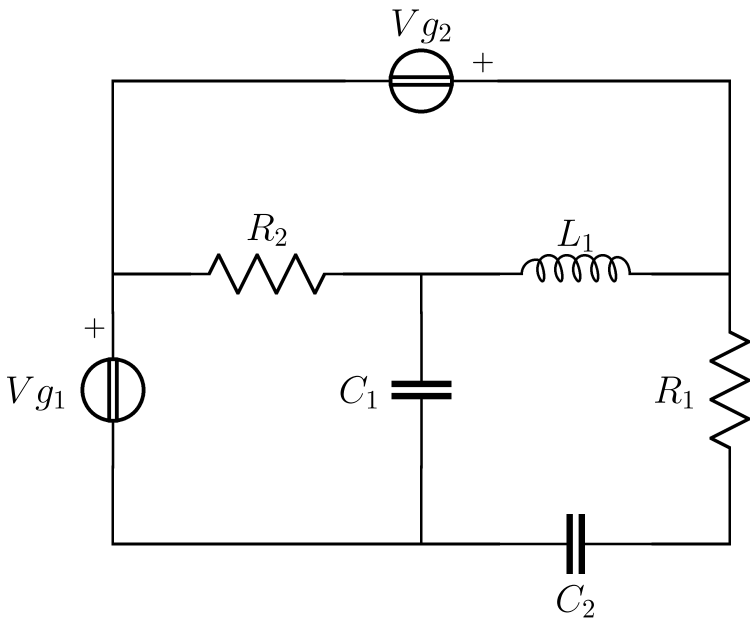
Esercizio ggcesame$_2015-02-10_A5$

Risolvere il circuito in figura



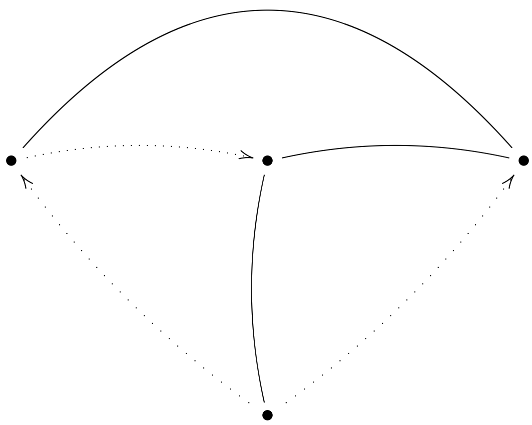
$\mathbf{V_{g_1}} = 3+4j$
$C_1 = 1$
$R_1 = \frac{1}{2}$
$C_2 = \frac{1}{2}$
$R_2 = 2$
$L_1 = 2$
$\mathbf{V_{g_2}} = -2-8j$
$\omega = 1$

Semplificazioni serie/parallelo

$Z_a = R_1 + \frac{1}{j\omega C_2} = \frac{1}{2} - 2j$
$Y_a = \frac{2}{17} + \frac{8}{17}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_2} & -j\omega L_1\mathbf{I_3} & = & \mathbf{V_{g1}}+\mathbf{V_{g2}} \\ (\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+Z_a+j\omega L_1)\mathbf{I_2} & -j\omega L_1\mathbf{I_3} & = & 0 \\ -j\omega L_1\mathbf{I_1} & -j\omega L_1\mathbf{I_2} & +(R_2+j\omega L_1)\mathbf{I_3} & = & -\mathbf{V_{g2}} \end{cases}$$

Sostituzione

$$\begin{cases} j\mathbf{I_1} & +j\mathbf{I_2} & -2j\mathbf{I_3} & = & 1-4j \\ j\mathbf{I_1} & +(\frac{1}{2}-j)\mathbf{I_2} & -2j\mathbf{I_3} & = & 0 \\ -2j\mathbf{I_1} & -2j\mathbf{I_2} & +(2+2j)\mathbf{I_3} & = & 2+8j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & j \\ \mathbf{I_2} & = & -2 \\ \mathbf{I_3} & = & 1+j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}}=\mathbf{I_1}=j & P_{c_{V_{g1}}}=\frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}}=2-\frac{3}{2}j \\ \mathbf{I_{V_{g2}}}=\mathbf{I_1}-\mathbf{I_3}=-1 & P_{c_{V_{g2}}}=\frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}}=1+4j \end{array}$$

$P_{c_{tot}}=3+\frac{5}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}}=\mathbf{I_2}=-2 & P_{a_{R_1}}=\frac{1}{2}R_1|\mathbf{I_{R_1}}|^2=1 \\ \mathbf{I_{R_2}}=\mathbf{I_3}=1+j & P_{a_{R_2}}=\frac{1}{2}R_2|\mathbf{I_{R_2}}|^2=2 \end{array}$$

$P_{a_{tot}}=3=\Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}}=(-\mathbf{I_1}-\mathbf{I_2})\frac{1}{j\omega C_1}=-1-2j & Q_{C_1}=-\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2=-\frac{5}{2} \\ \mathbf{I_{L_1}}=-\mathbf{I_1}-\mathbf{I_2}+\mathbf{I_3}=3 & Q_{L_1}=\frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2=9 \\ \mathbf{V_{C_2}}=\frac{\mathbf{I_2}}{j\omega C_2}=4j & Q_{C_2}=-\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2=-4 \end{array}$$

$Q_{tot}=\frac{5}{2}=\Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{g_1} = 3 + 4j;$	$I_{g_1} = j;$	$Pc_{V_{g1}} = 2 - \frac{3}{2}j$
$V_{C_1} = 1 + 2j;$	$I_{C_1} = 2 - j;$	$Q_{C_1} = -\frac{5}{2}$
$V_{R_1} + V_{C_2} = 1 - 4j;$	$I_{R_1} = I_{C_2} = -2;$	$Pa_{R_1} = 1$
$Q_{C_2} = -4$		
$V_{R_2} = -2 - 2j;$	$I_{R_2} = 1 + j;$	$Pa_{R_2} = 2$
$V_{L_1} = -6j;$	$I_{L_1} = 3;$	$Q_{L_1} = 9$
$V_{g_2} = -2 - 8j;$	$I_{g_2} = -1;$	$Pc_{V_{g2}} = 1 + 4j$