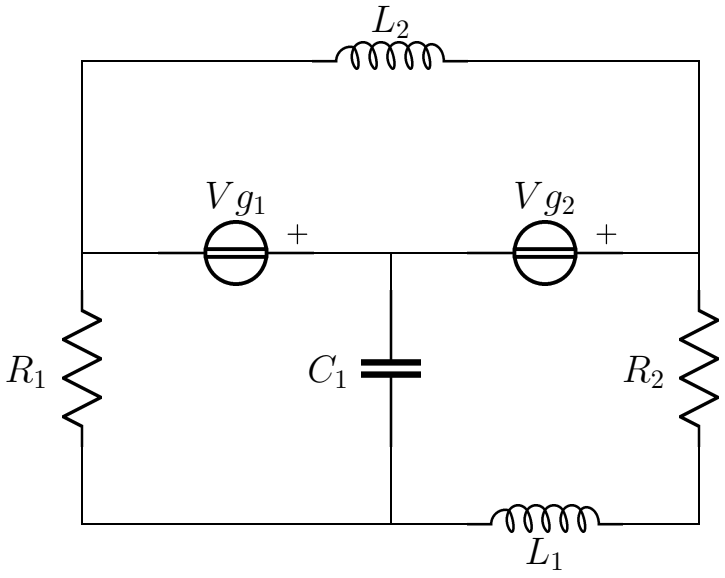
Esercizio ggcesame$_2$015 − 02 − 10$_A$3$_N$odi$_r$it2

Risolvere il circuito in figura



$R_1 = \frac{1}{2}$
$C_1 = 3$
$R_2 = 1$
$L_1 = 1$
$\mathbf{V_{g_1}} = 1 - j$
$\mathbf{V_{g_2}} = -1$
$L_2 = 1$
$\omega = 1$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_1 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{R_1} + \frac{1}{j\omega L_2})\mathbf{E_1} & -\frac{1}{j\omega L_2}\mathbf{E_2} & -\frac{1}{R_1}\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ -\frac{1}{j\omega L_2}\mathbf{E_1} & +(Y_a + \frac{1}{j\omega L_2})\mathbf{E_2} & -Y_a\mathbf{E_3} & = & \mathbf{I_{x_2}} \\ -\frac{1}{R_1}\mathbf{E_1} & -Y_a\mathbf{E_2} & +(\frac{1}{R_1} + j\omega C_1 + Y_a)\mathbf{E_3} & = & 0 \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_1}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (2-j)\mathbf{E_1} & +j\mathbf{E_2} & -2\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ j\mathbf{E_1} & +(\frac{1}{2} - \frac{3}{2}j)\mathbf{E_2} & +(-\frac{1}{2} + \frac{1}{2}j)\mathbf{E_3} & = & \mathbf{I_{x_2}} \\ -2\mathbf{E_1} & +(-\frac{1}{2} + \frac{1}{2}j)\mathbf{E_2} & +(\frac{5}{2} + \frac{5}{2}j)\mathbf{E_3} & = & 0 \\ -\mathbf{E_1} & & & = & 1-j \\ & \mathbf{E_2} & & = & -1 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} = -1+j \\ \mathbf{E_2} = -1 \\ \mathbf{E_3} = j \\ \mathbf{I_{x_1}} = 1 \\ \mathbf{I_{x_2}} = -2 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\mathbf{I_{V_{g1}}} = \mathbf{I_{x_2}} = 1 \quad P_{c_{V_{g1}}} = \tfrac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = \tfrac{1}{2} - \tfrac{1}{2}j$$
$$\mathbf{I_{V_{g2}}} = \mathbf{I_{x_2}} = -2 \quad P_{c_{V_{g2}}} = \tfrac{1}{2}\mathbf{V_{g2}}\mathbf{I}^*_{\mathbf{V_{g2}}} = 1$$

$P_{c_{tot}} = \frac{3}{2} - \frac{1}{2}j$

Potenza attiva assorbita dai resistori:

$$\mathbf{I_{R_1}} = \tfrac{\mathbf{E_1}-\mathbf{E_3}}{R_1} = -2 \quad P_{a_{R_1}} = \tfrac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1$$
$$\mathbf{I_{R_2}} = \tfrac{\mathbf{E_2}-\mathbf{E_3}}{Z_a} = -1 \quad P_{a_{R_2}} = \tfrac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \tfrac{1}{2}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\mathbf{V_{C_1}} = -\mathbf{E_3} = -j \quad Q_{C_1} = -\tfrac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\tfrac{3}{2}$$
$$\mathbf{I_{L_2}} = \tfrac{\mathbf{E_2}-\mathbf{E_1}}{j\omega L_2} = -1 \quad Q_{L_2} = \tfrac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \tfrac{1}{2}$$
$$\mathbf{I_{L_1}} = \tfrac{\mathbf{E_2}-\mathbf{E_3}}{Z_a} = -1 \quad Q_{L_1} = \tfrac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \tfrac{1}{2}$$

$Q_{tot} = -\frac{1}{2} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{R_1} = -1$; $I_{R_1} = 2$; $Pa_{R_1} = 1$
$V_{C_1} = -j$; $I_{C_1} = -3$; $Q_{C_1} = -\frac{3}{2}$
$V_{R_2} + V_{L_1} = -1 - j$; $I_{R_2} = I_{L_1} = 1$; $Pa_{R_2} = \frac{1}{2}$
$Q_{L_1} = \frac{1}{2}$
$V_{g_1} = 1 - j$; $I_{g_1} = 1$; $Pc_{V_{g1}} = \frac{1}{2} - \frac{1}{2}j$
$V_{g_2} = -1$; $I_{g_2} = -2$; $Pc_{V_{g2}} = 1$
$V_{L_2} = -j$; $I_{L_2} = 1$; $Q_{L_2} = \frac{1}{2}$