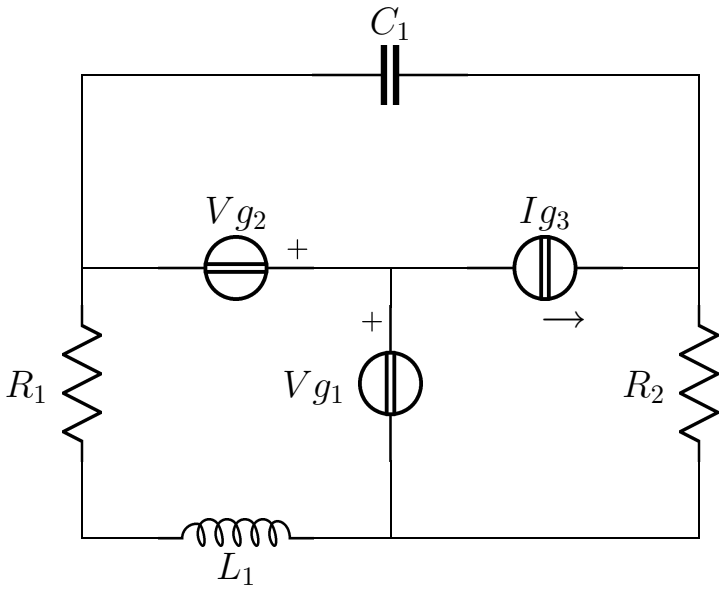
Esercizio $\mathbf{ggcesame_2015 - 02 - 10_A1_Nodi_rit2}$

Risolvere il circuito in figura



$R_1 = 1$
$L_1 = \frac{1}{2}$
$\mathbf{V_{g_1}} = j$
$R_2 = \frac{1}{2}$
$\mathbf{V_{g_2}} = -1$
$\mathbf{I_{g_3}} = 4 + 6j$
$C_1 = 2$
$\omega = 2$

Semplificazioni serie/parallelo

$Z_a = R_1 + j\omega L_1 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (Y_a + j\omega C_1)\mathbf{E_1} & -j\omega C_1\mathbf{E_2} & -Y_a\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ -j\omega C_1\mathbf{E_1} & +(\frac{1}{R_2} + j\omega C_1)\mathbf{E_2} & -\frac{1}{R_2}\mathbf{E_3} & = & \mathbf{I_{g_3}} \\ -Y_a\mathbf{E_1} & -\frac{1}{R_2}\mathbf{E_2} & +(Y_a + \frac{1}{R_2})\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2} + \frac{7}{2}j)\mathbf{E_1} & -4j\mathbf{E_2} & +(-\frac{1}{2} + \frac{1}{2}j)\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ -4j\mathbf{E_1} & +(2 + 4j)\mathbf{E_2} & -2\mathbf{E_3} & = & 4 + 6j \\ (-\frac{1}{2} + \frac{1}{2}j)\mathbf{E_1} & -2\mathbf{E_2} & +(\frac{5}{2} - \frac{1}{2}j)\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & j \\ -\mathbf{E_1} & & & = & -1 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} = 1 \\ \mathbf{E_2} = 2 \\ \mathbf{E_3} = -j \\ \mathbf{I_{x_1}} = 5+2j \\ \mathbf{I_{x_2}} = -1+4j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_2}} = 5+2j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = 1+\frac{5}{2}j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_2}} = -1+4j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I}^*_{\mathbf{V_{g2}}} = \frac{1}{2}+2j \\ \mathbf{V_{I_{g3}}} = \mathbf{E_2} = 2 & P_{c_{I_{g3}}} = \frac{1}{2}\mathbf{V_{I_{g3}}}\mathbf{I}^*_{\mathbf{g3}} = 4-6j \end{array}$$

$P_{c_{tot}} = \frac{11}{2} - \frac{3}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{Z_a} = 1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{R_2} = 4+2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 5 \end{array}$$

$P_{a_{tot}} = \frac{11}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{E_2} - \mathbf{E_1} = 1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -2 \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{Z_a} = 1 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -\frac{3}{2} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{R_1}+V_{L_1} = 1+j; & I_{R_1} = I_{L_1} = -1; & Pa_{R_1} = \frac{1}{2} \\ Q_{L_1} = \frac{1}{2} & & \\ V_{g_1} = j; & I_{g_1} = 5+2j; & Pc_{V_{g1}} = 1+\frac{5}{2}j \\ V_{R_2} = 2+j; & I_{R_2} = -4-2j; & Pa_{R_2} = 5 \\ V_{g_2} = -1; & I_{g_2} = -1+4j; & Pc_{V_{g2}} = \frac{1}{2}+2j \\ V_{g_3} = 2; & I_{g_3} = 4+6j; & Pc_{I_{g3}} = 4-6j \\ V_{C_1} = 1; & I_{C_1} = -4j; & Q_{C_1} = -2 \end{array}$$