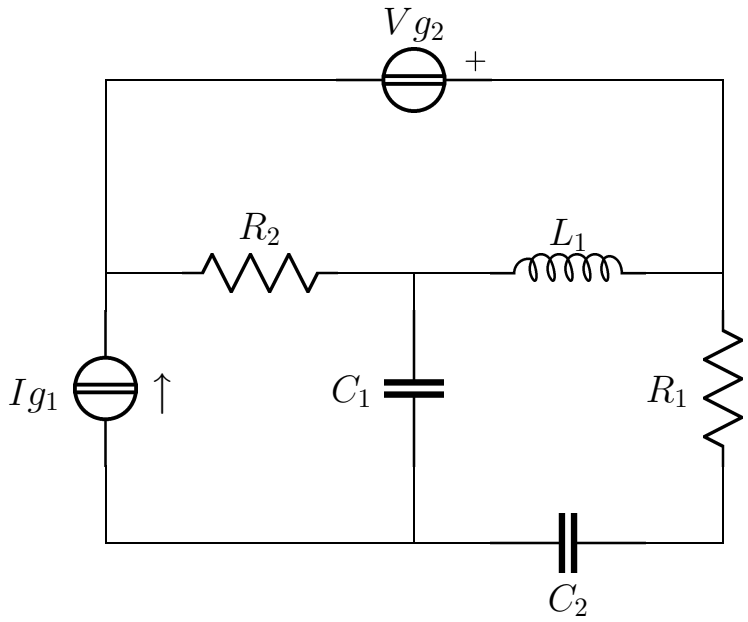
Esercizio $ggcesame_2015 - 02 - 10_A2_Nodi_rit3$

Risolvere il circuito in figura



$\mathbf{I_{g_1}} = j$
$C_1 = 1$
$R_1 = \frac{5}{4}$
$C_2 = \frac{1}{5}$
$R_2 = 1$
$L_1 = 1$
$\mathbf{V_{g_2}} = 1 - 8j$
$\omega = 2$

Semplificazioni serie/parallelo
$Z_a = R_1 + \frac{1}{j\omega C_2} = \frac{5}{4} - \frac{5}{2}j$
$Y_a = \frac{4}{25} + \frac{8}{25}j$

Risoluzione dell'esercizio con il metodo dei nodi
Sistema

$$\begin{cases} \frac{1}{R_2}\mathbf{E_1} & -\frac{1}{R_2}\mathbf{E_2} & & = & \mathbf{I_{g_1}} - \mathbf{I_{x_2}} \\ -\frac{1}{R_2}\mathbf{E_1} & +(j\omega C_1 + \frac{1}{R_2} + \frac{1}{j\omega L_1})\mathbf{E_2} & -j\omega C_1\mathbf{E_3} & = & 0 \\ & -j\omega C_1\mathbf{E_2} & +(j\omega C_1 + Y_a)\mathbf{E_3} & = & -\mathbf{I_{g_1}} \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} \mathbf{E_1} & -\mathbf{E_2} & & = & j - \mathbf{I_{x_2}} \\ -\mathbf{E_1} & +(1 + \frac{3}{2}j)\mathbf{E_2} & -2j\mathbf{E_3} & = & 0 \\ & -2j\mathbf{E_2} & +(\frac{4}{25} + \frac{58}{25}j)\mathbf{E_3} & = & -j \\ -\mathbf{E_1} & & & = & 1 - 8j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} &= -1+8j \\ \mathbf{E_2} &= -2+6j \\ \mathbf{E_3} &= -\frac{5}{2}+5j \\ \mathbf{I_{x_2}} &= -1-j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = \mathbf{E_1} - \mathbf{E_3} = \frac{3}{2}+3j & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{3}{2}-\frac{3}{4}j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_1}} = -1-j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = \frac{7}{2}+\frac{9}{2}j \end{array}$$

$P_{c_{tot}} = 5+\frac{15}{4}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{-\mathbf{E_3}}{Z_a} = 2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{5}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{R_2} = -1-2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{5}{2} \end{array}$$

$P_{a_{tot}} = 5 = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{E_2} - \mathbf{E_3} = \frac{1}{2}+j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{5}{4} \\ \mathbf{I_{L_1}} = \frac{-\mathbf{E_2}}{j\omega L_1} = -3-j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 10 \\ \mathbf{V_{C_2}} = \frac{-\mathbf{E_3}Y_a}{j\omega C_2} = -5j & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -5 \end{array}$$

$Q_{tot} = \frac{15}{4} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{g_1} = \frac{3}{2}+3j; & I_{g_1} = j; & Pc_{I_{g1}} = \frac{3}{2}-\frac{3}{4}j \\ V_{C_1} = \frac{1}{2}+j; & I_{C_1} = 2-j; & Q_{C_1} = -\frac{5}{4} \\ V_{R_1}+V_{C_2} = \frac{5}{2}-5j; & I_{R_1} = I_{C_2} = -2; & Pa_{R_1} = \frac{5}{2} \\ Q_{C_2} = -5 & & \\ V_{R_2} = -1-2j; & I_{R_2} = 1+2j; & Pa_{R_2} = \frac{5}{2} \\ V_{L_1} = 2-6j; & I_{L_1} = 3+j; & Q_{L_1} = 10 \\ V_{g_2} = 1-8j; & I_{g_2} = -1-j; & Pc_{V_{g2}} = \frac{7}{2}+\frac{9}{2}j \end{array}$$