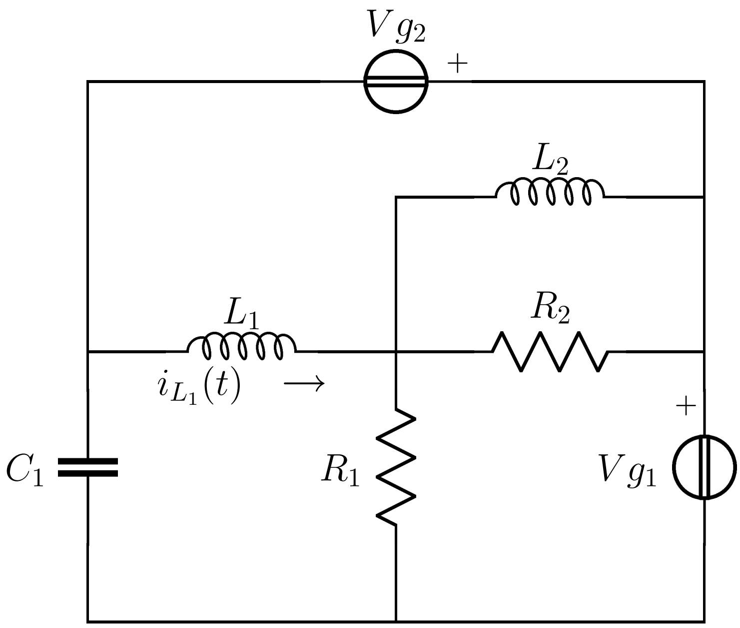
Esercizio ggcesame$_{2}016-01-29_{P}arallelo2_{M}aglie$

Risolvere il circuito in figura



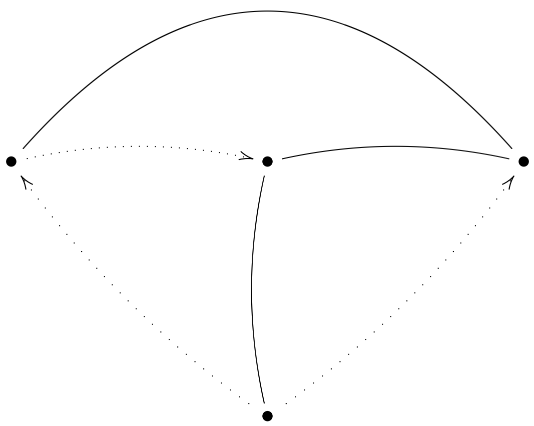
$C_1 = \frac{1}{2}$
$R_1 = 1$
$\mathbf{V_{g_1}} = 3$
$L_1 = 2$
$R_2 = 1$
$L_2 = 1$
$\mathbf{V_{g_2}} = 3 - 2j$
$\omega = 1$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_2} + \frac{1}{j\omega L_2} = 1 - j$
$Z_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (\frac{1}{j\omega C_1} + R_1 + Z_a)\mathbf{I_1} & +(R_1 + Z_a)\mathbf{I_2} & -Z_a\mathbf{I_3} & = & \mathbf{V_{g_2}} \\ (R_1 + Z_a)\mathbf{I_1} & +(R_1 + Z_a)\mathbf{I_2} & -Z_a\mathbf{I_3} & = & \mathbf{V_{g_1}} \\ -Z_a\mathbf{I_1} & -Z_a\mathbf{I_2} & +(j\omega L_1 + Z_a)\mathbf{I_3} & = & -\mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{3}{2} - \frac{3}{2}j)\mathbf{I_1} & +(\frac{3}{2} + \frac{1}{2}j)\mathbf{I_2} & +(-\frac{1}{2} - \frac{1}{2}j)\mathbf{I_3} & = & 3 - 2j \\ (\frac{3}{2} + \frac{1}{2}j)\mathbf{I_1} & +(\frac{3}{2} + \frac{1}{2}j)\mathbf{I_2} & +(-\frac{1}{2} - \frac{1}{2}j)\mathbf{I_3} & = & 3 \\ (-\frac{1}{2} - \frac{1}{2}j)\mathbf{I_1} & +(-\frac{1}{2} - \frac{1}{2}j)\mathbf{I_2} & +(\frac{1}{2} + \frac{5}{2}j)\mathbf{I_3} & = & -3 + 2j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & 1 \\ \mathbf{I_2} & = & 1 \\ \mathbf{I_3} & = & 1 + j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_2} = 1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = \frac{3}{2} \\ \mathbf{I_{V_{g2}}} = \mathbf{I_1} - \mathbf{I_3} = -j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 1 + \frac{3}{2}j \end{array}$$

$P_{c_{tot}} = \frac{5}{2} + \frac{3}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = -\mathbf{I_1} - \mathbf{I_2} = -2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 2 \\ \mathbf{I_{R_2}} = \frac{(-\mathbf{I_1} - \mathbf{I_2} + \mathbf{I_3})Z_a}{R_2} = -1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{5}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{I_1}\frac{1}{j\omega C_1} = -2j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -1 \\ \mathbf{I_{L_1}} = \mathbf{I_3} = 1 + j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 2 \\ \mathbf{I_{L_2}} = \frac{(-\mathbf{I_1} - \mathbf{I_2} + \mathbf{I_3})Z_a}{j\omega L_2} = j & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = \frac{3}{2} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{L_1}} = \mathbf{I_3} = 1 + j$$

$$i_{L_1}(t) = \sqrt{2}\cos(t + \frac{\pi}{4})$$

Soluzioni:

$V_{C_1} = 2j;$	$I_{C_1} = 1;$	$Q_{C_1} = -1$
$V_{R_1} = 2;$	$I_{R_1} = -2;$	$Pa_{R_1} = 2$
$V_{g_1} = 3;$	$I_{g_1} = 1;$	$Pc_{V_{g1}} = \frac{3}{2}$
$V_{L_1} = 2 - 2j;$	$I_{L_1} = 1 + j;$	$Q_{L_1} = 2$
$V_{R_2} = V_{L_2} = 1;$	$I_{R_2} + I_{L_2} = -1 + j;$	$Pa_{R_2} = \frac{1}{2}$
$Q_{L_2} = \frac{1}{2}$		
$V_{g_2} = 3 - 2j;$	$I_{g_2} = -j;$	$Pc_{V_{g2}} = 1 + \frac{3}{2}j$