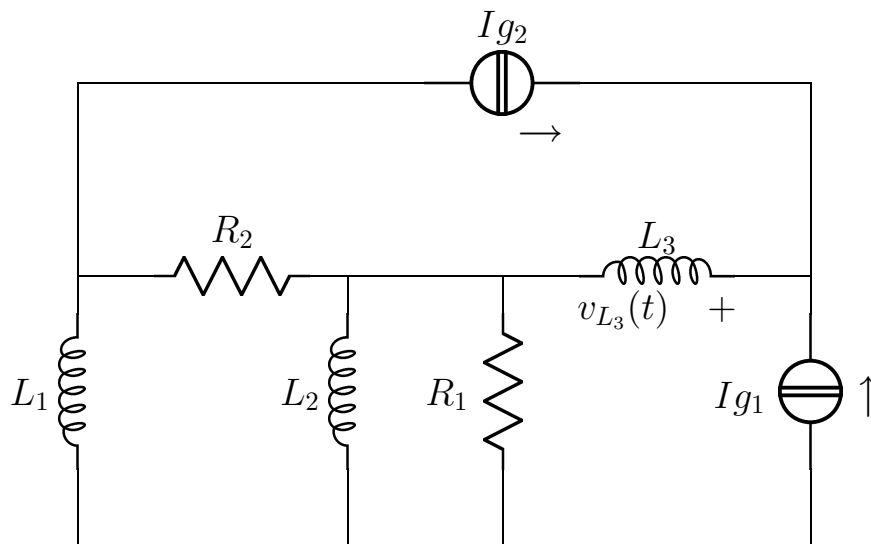


Esercizio ggcesame₂₀₁₆ – 01 – 29_{parallelo4}; Maglie

Risolvere il circuito in figura



$$\begin{aligned} L_1 &= 1 \\ R_1 &= 1 \\ L_2 &= \frac{1}{2} \\ \mathbf{I}_{g1} &= -1 + j \\ R_2 &= 2 \\ L_3 &= 2 \\ \mathbf{I}_{g2} &= -1 + j \\ \omega &= 1 \end{aligned}$$

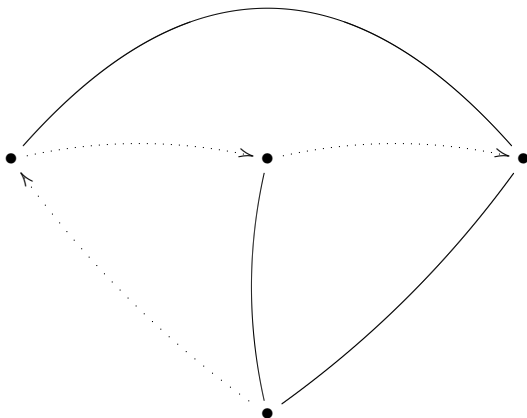
Semplificazioni serie/parallelo

$$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_2} = 1 - 2j$$

$$Z_a = \frac{1}{\frac{1}{5}} + \frac{2}{5}j$$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\left\{ \begin{array}{rcl} j\omega L_1 \mathbf{I}_1 & = & -\mathbf{V}_{x_1} + \mathbf{V}_{x_2} \\ (Z_a + R_2) \mathbf{I}_2 & - Z_a \mathbf{I}_3 & = \mathbf{V}_{x_1} - \mathbf{V}_{x_2} \\ -Z_a \mathbf{I}_2 & + (Z_a + j\omega L_3) \mathbf{I}_3 & = -\mathbf{V}_{x_1} \\ -\mathbf{I}_1 & + \mathbf{I}_2 & - \mathbf{I}_3 = \mathbf{I}_{g_1} \\ \mathbf{I}_1 & - \mathbf{I}_2 & = \mathbf{I}_{g_2} \end{array} \right.$$

Sostituzione

$$\left\{ \begin{array}{rcl} j\mathbf{I}_1 & = & -\mathbf{V}_{x_1} + \mathbf{V}_{x_2} \\ (\frac{11}{5} + \frac{2}{5}j)\mathbf{I}_2 & + (-\frac{1}{5} - \frac{2}{5}j)\mathbf{I}_3 & = \mathbf{V}_{x_1} - \mathbf{V}_{x_2} \\ (-\frac{1}{5} - \frac{2}{5}j)\mathbf{I}_2 & + (\frac{1}{5} + \frac{12}{5}j)\mathbf{I}_3 & = -\mathbf{V}_{x_1} \\ -\mathbf{I}_1 & + \mathbf{I}_2 & - \mathbf{I}_3 = -1 + j \\ \mathbf{I}_1 & - \mathbf{I}_2 & = -1 + j \end{array} \right.$$

Soluzione

$$\left\{ \begin{array}{rcl} \mathbf{I}_1 & = & j \\ \mathbf{I}_2 & = & 1 \\ \mathbf{I}_3 & = & 2 - 2j \\ \mathbf{V}_{x_1} & = & -5 - 4j \\ \mathbf{V}_{x_2} & = & -6 - 4j \end{array} \right.$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{aligned} \mathbf{V}_{\mathbf{I}_{g1}} = \mathbf{V}_{\mathbf{x}_{g1}} = -5 - 4j \quad P_{c_{I_{g1}}} &= \frac{1}{2} \mathbf{V}_{\mathbf{I}_{g1}} \mathbf{I}_{g1}^* = \frac{1}{2} + \frac{9}{2}j \\ \mathbf{V}_{\mathbf{I}_{g2}} = \mathbf{V}_{\mathbf{x}_{g1}} = -6 - 4j \quad P_{c_{I_{g2}}} &= \frac{1}{2} \mathbf{V}_{\mathbf{I}_{g2}} \mathbf{I}_{g2}^* = 1 + 5j \\ P_{c_{tot}} &= \frac{3}{2} + \frac{19}{2}j \end{aligned}$$

Potenza attiva assorbita dai resistori:

$$\begin{aligned} \mathbf{I}_{\mathbf{R}_1} &= \frac{(-\mathbf{I}_2 + \mathbf{I}_3)Z_a}{R_1} = 1 \quad P_{a_{R_1}} = \frac{1}{2} R_1 |\mathbf{I}_{\mathbf{R}_1}|^2 = \frac{1}{2} \\ \mathbf{I}_{\mathbf{R}_2} &= \mathbf{I}_2 = 1 \quad P_{a_{R_2}} = \frac{1}{2} R_2 |\mathbf{I}_{\mathbf{R}_2}|^2 = 1 \\ P_{a_{tot}} &= \frac{3}{2} = \Re\{P_{c_{tot}}\} \end{aligned}$$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{aligned} \mathbf{I}_{L_1} &= \mathbf{I}_1 = j \quad Q_{L_1} = \frac{1}{2} \omega L_1 |\mathbf{I}_{L_1}|^2 = \frac{1}{2} \\ \mathbf{I}_{L_3} &= -\mathbf{I}_3 = -2 + 2j \quad Q_{L_3} = \frac{1}{2} \omega L_3 |\mathbf{I}_{L_3}|^2 = 8 \\ \mathbf{I}_{L_2} &= \frac{(-\mathbf{I}_2 + \mathbf{I}_3)Z_a}{j\omega L_2} = -2j \quad Q_{L_2} = \frac{1}{2} \omega L_2 |\mathbf{I}_{L_2}|^2 = 1 \end{aligned}$$

$$Q_{tot} = \frac{19}{2} = \Im\{P_{c_{tot}}\}$$

Calcolo tensioni e correnti

$$\mathbf{V}_{L_3} = -\mathbf{I}_3 j\omega L_3 = -4 - 4j$$

$$v_{L_3}(t) = 4\sqrt{2} \cos\left(t + \frac{5\pi}{4}\right)$$

Soluzioni:

$$\begin{array}{lll} V_{L_1} = 1; & I_{L_1} = j; & Q_{L_1} = \frac{1}{2} \\ V_{R_1} = V_{L_2} = -1; & I_{R_1} + I_{L_2} = 1 - 2j; & Pa_{R_1} = \frac{1}{2} \\ Q_{L_2} = 1 & & \\ V_{g_1} = -5 - 4j; & I_{g_1} = -1 + j; & Pc_{I_{g_1}} = \frac{1}{2} + \frac{9}{2}j \\ V_{R_2} = -2; & I_{R_2} = 1; & Pa_{R_2} = 1 \\ V_{L_3} = -4 - 4j; & I_{L_3} = 2 - 2j; & Q_{L_3} = 8 \\ V_{g_2} = -6 - 4j; & I_{g_2} = -1 + j; & Pc_{I_{g_2}} = 1 + 5j \end{array}$$