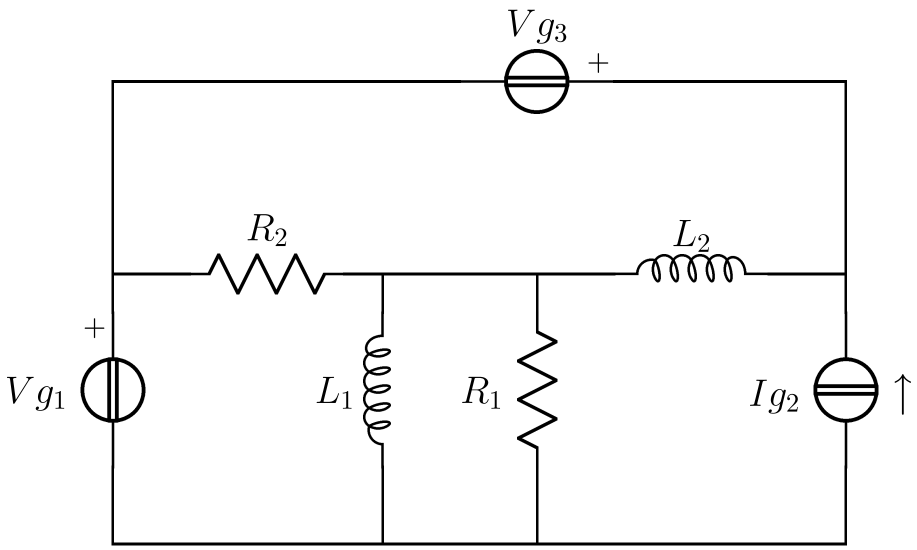
Esercizio 2013-07-02$_{Nodi_Rif1_A2_Fasori}$

Risolvere il circuito in figura



$\mathbf{V_{g_1}} = -5 + 4j$
$R_1 = 1$
$L_1 = \frac{1}{2}$
$\mathbf{I_{g_2}} = j$
$R_2 = 2$
$L_2 = 2$
$\mathbf{V_{g_3}} = 4 - 2j$
$\omega = 1$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_1} = 1 - 2j$
$Z_a = \frac{1}{5} + \frac{2}{5}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\left\{\begin{array}{rrrcr}
(Y_a + \frac{1}{R_2} + \frac{1}{j\omega L_2})\mathbf{E_1} & -\frac{1}{j\omega L_2}\mathbf{E_2} & -Y_a\mathbf{E_3} & = & 0 \\
-\frac{1}{j\omega L_2}\mathbf{E_1} & +\frac{1}{j\omega L_2}\mathbf{E_2} & & = & \mathbf{I_{g_2}} + \mathbf{I_{x_3}} \\
-Y_a\mathbf{E_1} & & +Y_a\mathbf{E_3} & = & -\mathbf{I_{g_2}} - \mathbf{I_{x_1}} \\
& & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\
& \mathbf{E_2} & & = & \mathbf{V_{g_3}}
\end{array}\right.$$

Sostituzione

$$\left\{\begin{array}{rrrcr}
(\frac{3}{2} - \frac{5}{2}j)\mathbf{E_1} & +\frac{1}{2}j\mathbf{E_2} & +(-1+2j)\mathbf{E_3} & = & 0 \\
\frac{1}{2}j\mathbf{E_1} & -\frac{1}{2}j\mathbf{E_2} & & = & j + \mathbf{I_{x_3}} \\
(-1+2j)\mathbf{E_1} & & +(1-2j)\mathbf{E_3} & = & -j - \mathbf{I_{x_1}} \\
& & -\mathbf{E_3} & = & -5 + 4j \\
& \mathbf{E_2} & & = & 4 - 2j
\end{array}\right.$$

Soluzione

$$\begin{cases} \mathbf{E_1} &= 4-4j \\ \mathbf{E_2} &= 4-2j \\ \mathbf{E_3} &= 5-4j \\ \mathbf{I_{x_1}} &= -1+j \\ \mathbf{I_{x_3}} &= 1-j \end{cases}$$

Bilancio di potenza
Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_g2}} = -1+j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = \frac{9}{2}+\frac{1}{2}j \\ \mathbf{V_{I_{g2}}} = \mathbf{E_2} - \mathbf{E_3} = -1+2j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = 1+\frac{1}{2}j \\ \mathbf{I_{V_{g3}}} = \mathbf{I_{x_g2}} = 1-j & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I^*_{V_{g3}}} = 3+j \end{array}$$

$P_{c_{tot}} = \frac{17}{2} + 2j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{R_1} = -1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_1}}{R_2} = 2-2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 8 \end{array}$$

$P_{a_{tot}} = \frac{17}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{j\omega L_2} = 1 & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = 1 \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{j\omega L_1} = 2j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 1 \end{array}$$

$Q_{tot} = 2 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{g_1} = -5+4j; & I_{g_1} = -1+j; & Pc_{V_{g1}} = \frac{9}{2}+\frac{1}{2}j \\ V_{R_1} = V_{L_1} = -1; & I_{R_1} + I_{L_1} = 1-2j; & Pa_{R_1} = \frac{1}{2} \\ Q_{L_1} = 1 & & \\ V_{g_2} = -1+2j; & I_{g_2} = j; & Pc_{I_{g2}} = 1+\frac{1}{2}j \\ V_{R_2} = 4-4j; & I_{R_2} = -2+2j; & Pa_{R_2} = 8 \\ V_{L_2} = 2j; & I_{L_2} = -1; & Q_{L_2} = 1 \\ V_{g_3} = 4-2j; & I_{g_3} = 1-j; & Pc_{V_{g3}} = 3+j \end{array}$$