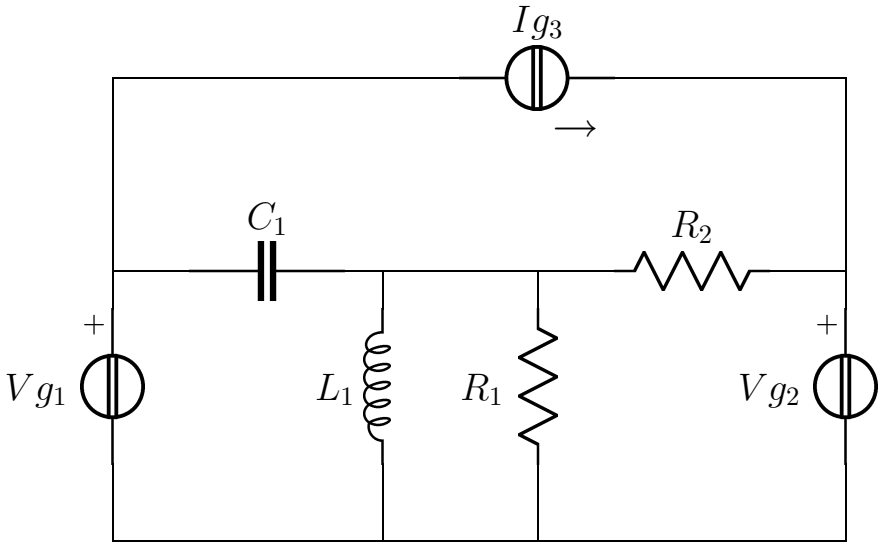
Esercizio ggcesame$_2$015 − 02 − 10$_B$1$_N$odi$_r$if4

Risolvere il circuito in figura



$\mathbf{V_{g_1}} = 2$
$R_1 = 1$
$L_1 = \frac{1}{2}$
$\mathbf{V_{g_2}} = 3 - 4j$
$C_1 = \frac{1}{2}$
$R_2 = 2$
$\mathbf{I_{g_3}} = 1 - j$
$\omega = 2$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_1} = 1 - j$
$Z_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} j\omega C_1\mathbf{E_1} & -j\omega C_1\mathbf{E_2} & & = & -\mathbf{I_{g_3}} + \mathbf{I_{x_1}} \\ -j\omega C_1\mathbf{E_1} & +(Y_a + j\omega C_1 + \frac{1}{R_2})\mathbf{E_2} & -\frac{1}{R_2}\mathbf{E_3} & = & 0 \\ & -\frac{1}{R_2}\mathbf{E_2} & +\frac{1}{R_2}\mathbf{E_3} & = & \mathbf{I_{g_3}} + \mathbf{I_{x_2}} \\ \mathbf{E_1} & & & = & \mathbf{V_{g_1}} \\ & & \mathbf{E_3} & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} j\mathbf{E_1} & -j\mathbf{E_2} & & = & -1 + j + \mathbf{I_{x_1}} \\ -j\mathbf{E_1} & +\frac{3}{2}\mathbf{E_2} & -\frac{1}{2}\mathbf{E_3} & = & 0 \\ & -\frac{1}{2}\mathbf{E_2} & +\frac{1}{2}\mathbf{E_3} & = & 1 - j + \mathbf{I_{x_2}} \\ \mathbf{E_1} & & & = & 2 \\ & & \mathbf{E_3} & = & 3 - 4j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} = 2 \\ \mathbf{E_2} = 1 \\ \mathbf{E_3} = 3-4j \\ \mathbf{I_{x_1}} = 1 \\ \mathbf{I_{x_2}} = -j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_g 2}} = 1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = 1 \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_g 2}} = -j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I}^*_{\mathbf{V_{g2}}} = 2+\frac{3}{2}j \\ \mathbf{V_{I_{g3}}} = \mathbf{E_3} - \mathbf{E_1} = 1-4j & P_{c_{I_{g3}}} = \frac{1}{2}\mathbf{V_{I_{g3}}}\mathbf{I}^*_{\mathbf{g3}} = \frac{5}{2}-\frac{3}{2}j \end{array}$$

$P_{c_{tot}} = \frac{11}{2}$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_2}}{R_1} = 1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_3}-\mathbf{E_2}}{R_2} = 1-2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 5 \end{array}$$

$P_{a_{tot}} = \frac{11}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{E_2} - \mathbf{E_1} = -1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{1}{2} \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_2}}{j\omega L_1} = -j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = 0 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{g_1} = 2; & I_{g_1} = 1; & Pc_{V_{g1}} = 1 \\ V_{R_1} = V_{L_1} = 1; & I_{R_1} + I_{L_1} = -1+j; & Pa_{R_1} = \frac{1}{2} \\ Q_{L_1} = \frac{1}{2} & & \\ V_{g_2} = 3-4j; & I_{g_2} = -j; & Pc_{V_{g2}} = 2+\frac{3}{2}j \\ V_{C_1} = -1; & I_{C_1} = j; & Q_{C_1} = -\frac{1}{2} \\ V_{R_2} = 2-4j; & I_{R_2} = -1+2j; & Pa_{R_2} = 5 \\ V_{g_3} = 1-4j; & I_{g_3} = 1-j; & Pc_{I_{g3}} = \frac{5}{2}-\frac{3}{2}j \end{array}$$