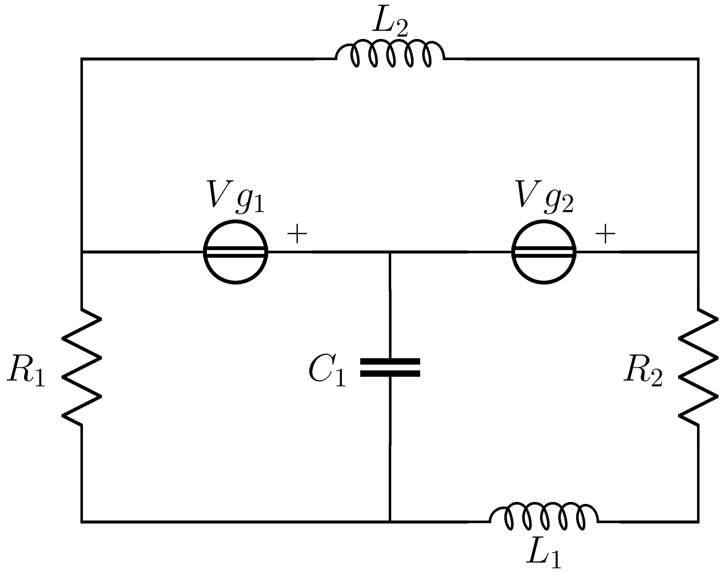
Esercizio ggcesame$_2 015 - 02 - 10_A 3_M aglie$

Risolvere il circuito in figura



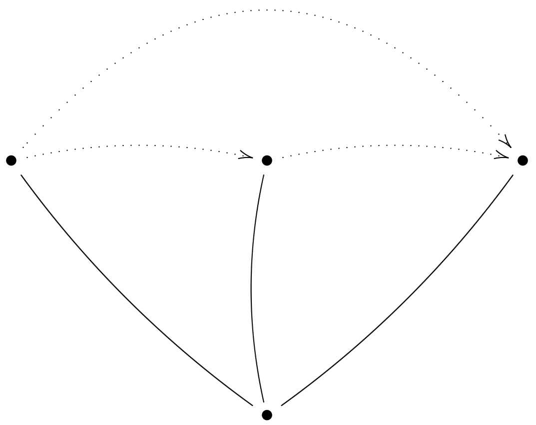
$R_1 = \frac{1}{2}$
$C_1 = 3$
$R_2 = 1$
$L_1 = 1$
$\mathbf{V_{g_1}} = 1 - j$
$\mathbf{V_{g_2}} = -1$
$L_2 = 1$
$\omega = 1$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_1 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (R_1 + \frac{1}{j\omega C_1})\mathbf{I_1} & -\frac{1}{j\omega C_1}\mathbf{I_2} & +R_1\mathbf{I_3} & = & \mathbf{V_{g1}} \\ -\frac{1}{j\omega C_1}\mathbf{I_1} & +(\frac{1}{j\omega C_1} + Z_a)\mathbf{I_2} & +Z_a\mathbf{I_3} & = & \mathbf{V_{g2}} \\ R_1\mathbf{I_1} & +Z_a\mathbf{I_2} & +(R_1 + Z_a + j\omega L_2)\mathbf{I_3} & = & 0 \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2} - \frac{1}{3}j)\mathbf{I_1} & +\frac{1}{3}j\mathbf{I_2} & +\frac{1}{2}\mathbf{I_3} & = & 1 - j \\ \frac{1}{3}j\mathbf{I_1} & +(1 + \frac{2}{3}j)\mathbf{I_2} & +(1 + j)\mathbf{I_3} & = & -1 \\ \frac{1}{2}\mathbf{I_1} & +(1 + j)\mathbf{I_2} & +(\frac{3}{2} + 2j)\mathbf{I_3} & = & 0 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & 1 \\ \mathbf{I_2} & = & -2 \\ \mathbf{I_3} & = & 1 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_1} = 1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = \frac{1}{2} - \frac{1}{2}j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_2} = -2 & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 1 \end{array}$$

$P_{c_{tot}} = \frac{3}{2} - \frac{1}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_1} + \mathbf{I_3} = 2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = -\mathbf{I_2} - \mathbf{I_3} = 1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = (-\mathbf{I_1} + \mathbf{I_2})\frac{1}{j\omega C_1} = j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{3}{2} \\ \mathbf{I_{L_2}} = \mathbf{I_3} = 1 & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{1}{2} \\ \mathbf{I_{L_1}} = -\mathbf{I_2} - \mathbf{I_3} = 1 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -\frac{1}{2} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$
\begin{array}{lll}
V_{R_1} = -1; & I_{R_1} = 2; & Pa_{R_1} = 1 \\
V_{C_1} = -j; & I_{C_1} = -3; & Q_{C_1} = -\frac{3}{2} \\
V_{R_2} + V_{L_1} = -1 - j; & I_{R_2} = I_{L_1} = 1; & Pa_{R_2} = \frac{1}{2} \\
Q_{L_1} = \frac{1}{2} & & \\
V_{g_1} = 1 - j; & I_{g_1} = 1; & Pc_{V_{g1}} = \frac{1}{2} - \frac{1}{2}j \\
V_{g_2} = -1; & I_{g_2} = -2; & Pc_{V_{g2}} = 1 \\
V_{L_2} = -j; & I_{L_2} = 1; & Q_{L_2} = \frac{1}{2}
\end{array}
$$