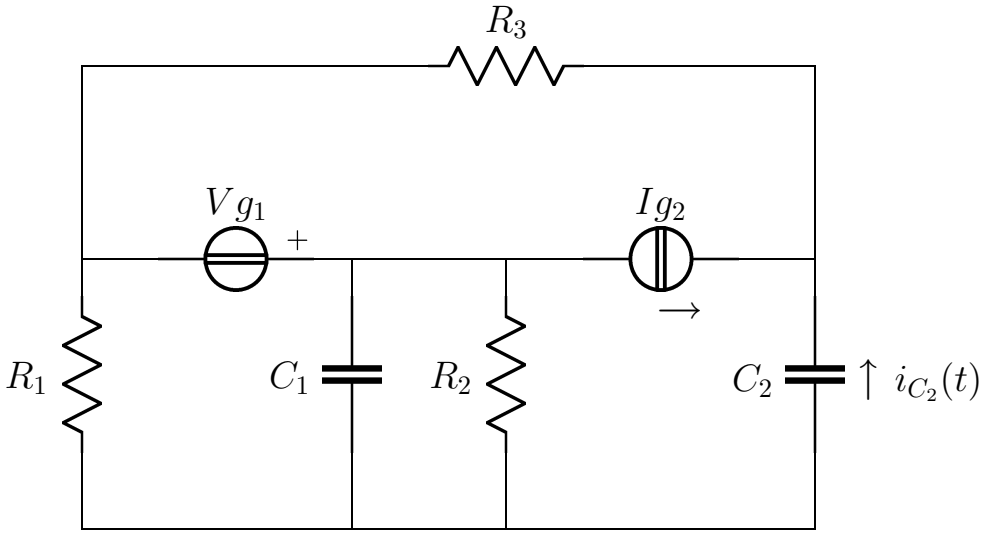
Esercizio ggcesame$_2$016 − 01 − 29$_P$arallelo1$_N$odiRif1

Risolvere il circuito in figura



$R_1 = \frac{1}{2}$
$R_2 = \frac{1}{2}$
$C_1 = 4$
$C_2 = 2$
$\mathbf{V_{g_1}} = -2$
$\mathbf{I_{g_2}} = 1 + 8j$
$R_3 = 1$
$\omega = 2$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_2} + j\omega C_1 = 2 + 8j$
$Z_a = \frac{1}{34} - \frac{2}{17}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} Y_a\mathbf{E_1} & & -Y_a\mathbf{E_3} & = & -\mathbf{I_{g_2}} + \mathbf{I_{x_1}} \\ & (j\omega C_2 + \frac{1}{R_3})\mathbf{E_2} & -j\omega C_2\mathbf{E_3} & = & \mathbf{I_{g_2}} \\ -Y_a\mathbf{E_1} & -j\omega C_2\mathbf{E_2} & +(\frac{1}{R_1} + Y_a + j\omega C_2)\mathbf{E_3} & = & 0 \\ \mathbf{E_1} & & & = & \mathbf{V_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} (2+8j)\mathbf{E_1} & & +(-2-8j)\mathbf{E_3} & = & -1-8j+\mathbf{I_{x_1}} \\ & (1+4j)\mathbf{E_2} & -4j\mathbf{E_3} & = & 1+8j \\ (-2-8j)\mathbf{E_1} & -4j\mathbf{E_2} & +(4+12j)\mathbf{E_3} & = & 0 \\ \mathbf{E_1} & & & = & -2 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} &= -2 \\ \mathbf{E_2} &= 1 \\ \mathbf{E_3} &= -1 \\ \mathbf{I_{x_1}} &= -1 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_2}} = -1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = 1 \\ \mathbf{V_{I_{g2}}} = \mathbf{E_2} - \mathbf{E_1} = 3 & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = \frac{3}{2} - 12j \end{array}$$

$P_{c_{tot}} = \frac{5}{2} - 12j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{-\mathbf{E_3}}{R_1} = 2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{R_2} = -2 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 1 \\ \mathbf{I_{R_3}} = \frac{\mathbf{E_2}}{R_3} = 1 & P_{a_{R_3}} = \frac{1}{2}R_3|\mathbf{I_{R_3}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{5}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_2}} = \mathbf{E_3} - \mathbf{E_2} = -2 & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -8 \\ \mathbf{V_{C_1}} = \mathbf{E_1} - \mathbf{E_3} = -1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -4 \end{array}$$

$Q_{tot} = -12 = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{C_2}} = (\mathbf{E_3} - \mathbf{E_2})j\omega C_2 = -8j$$

$$i_{C_2}(t) = 8\cos(2t - \frac{\pi}{2})$$

Soluzioni:

$V_{R_1} = 1;$	$I_{R_1} = -2;$	$Pa_{R_1} = 1$
$V_{R_2} = V_{C_1} = -1;$	$I_{R_2} + I_{C_1} = 2 + 8j;$	$Pa_{R_2} = 1$
$Q_{C_1} = -4$		
$V_{C_2} = 2;$	$I_{C_2} = -8j;$	$Q_{C_2} = -8$
$V_{g_1} = -2;$	$I_{g_1} = -1;$	$Pc_{V_{g1}} = 1$
$V_{g_2} = 3;$	$I_{g_2} = 1 + 8j;$	$Pc_{I_{g2}} = \frac{3}{2} - 12j$
$V_{R_3} = 1;$	$I_{R_3} = -1;$	$Pa_{R_3} = \frac{1}{2}$