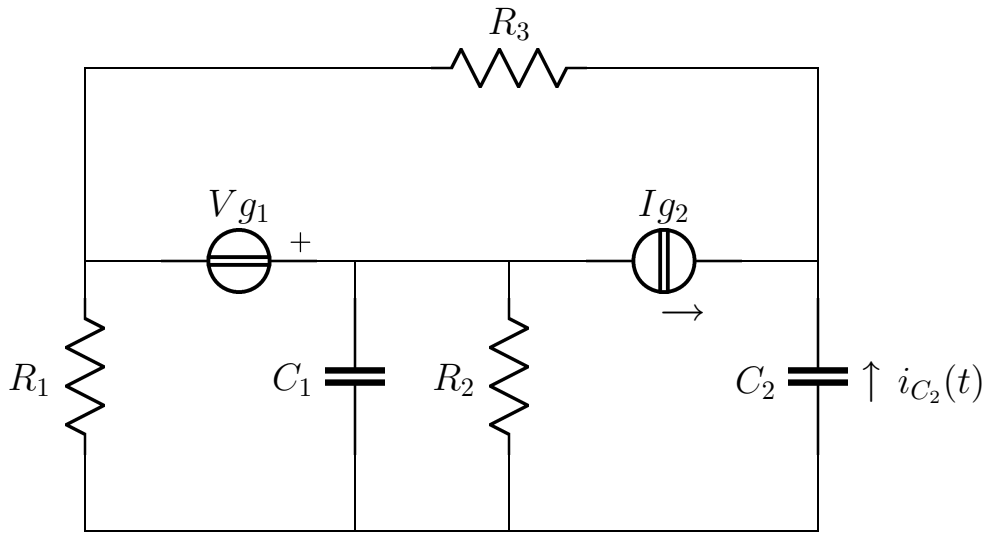
Esercizio ggcesame$_2$016 − 01 − 29$_P$arallelo1$_M$aglie

Risolvere il circuito in figura



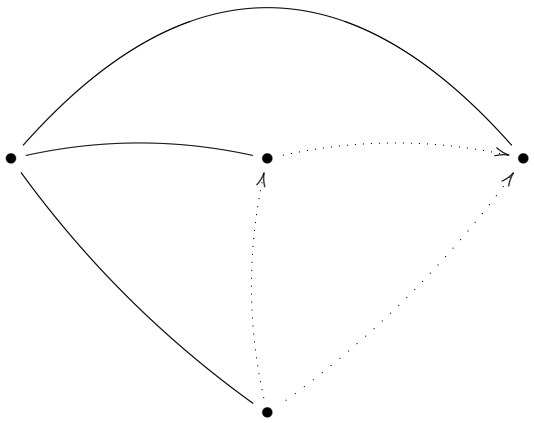
$R_1 = \frac{1}{2}$
$R_2 = \frac{1}{2}$
$C_1 = 4$
$C_2 = 2$
$\mathbf{V_{g_1}} = -2$
$\mathbf{I_{g_2}} = 1 + 8j$
$R_3 = 1$
$\omega = 2$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_2} + j\omega C_1 = 2 + 8j$
$Z_a = \frac{1}{34} - \frac{2}{17}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (R_1+Z_a)\mathbf{I_1} & & +R_1\mathbf{I_2} & & = & -\mathbf{V_{g_1}} \\ R_1\mathbf{I_1} & +(R_1+\frac{1}{j\omega C_2}+R_3)\mathbf{I_2} & +R_3\mathbf{I_3} & = & 0 \\ & & R_3\mathbf{I_2} & +R_3\mathbf{I_3} & = & \mathbf{V_{g_1}}+\mathbf{V_{x_2}} \\ & & & \mathbf{I_3} & = & \mathbf{I_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{9}{17}-\frac{2}{17}j)\mathbf{I_1} & & +\frac{1}{2}\mathbf{I_2} & & = & 2 \\ \frac{1}{2}\mathbf{I_1} & +(\frac{3}{2}-\frac{1}{4}j)\mathbf{I_2} & +\mathbf{I_3} & = & 0 \\ & & \mathbf{I_2} & +\mathbf{I_3} & = & -2+\mathbf{V_{x_2}} \\ & & & \mathbf{I_3} & = & 1+8j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & 2+8j \\ \mathbf{I_2} & = & -8j \\ \mathbf{I_3} & = & 1+8j \\ \mathbf{V_{x_2}} & = & 3 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = -\mathbf{I_1}+\mathbf{I_3} = -1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = 1 \\ \mathbf{V_{I_{g2}}} = \mathbf{V_{x_2}} = 3 & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = \frac{3}{2}-12j \end{array}$$

$P_{c_{tot}} = \frac{5}{2}-12j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = -\mathbf{I_1}-\mathbf{I_2} = -2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \frac{\mathbf{I_1}Z_a}{R_2} = 2 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 1 \\ \mathbf{I_{R_3}} = -\mathbf{I_2}-\mathbf{I_3} = -1 & P_{a_{R_3}} = \frac{1}{2}R_3|\mathbf{I_{R_3}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{5}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_2}} = \mathbf{I_2}\frac{1}{j\omega C_2} = -2 & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -8 \\ \mathbf{V_{C_1}} = \mathbf{I_1}Z_a = 1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -4 \end{array}$$

$Q_{tot} = -12 = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$\mathbf{I_{C_2}} = \mathbf{I_2} = -8j$

$i_{C_2}(t) = 8\cos(2t - \frac{\pi}{2})$

Soluzioni:

$V_{R_1} = 1;$	$I_{R_1} = -2;$	$Pa_{R_1} = 1$
$V_{R_2} = V_{C_1} = -1;$	$I_{R_2} + I_{C_1} = 2 + 8j;$	$Pa_{R_2} = 1$
$Q_{C_1} = -4$		
$V_{C_2} = 2;$	$I_{C_2} = -8j;$	$Q_{C_2} = -8$
$V_{g_1} = -2;$	$I_{g_1} = -1;$	$Pc_{V_{g1}} = 1$
$V_{g_2} = 3;$	$I_{g_2} = 1 + 8j;$	$Pc_{I_{g2}} = \frac{3}{2} - 12j$
$V_{R_3} = 1;$	$I_{R_3} = -1;$	$Pa_{R_3} = \frac{1}{2}$