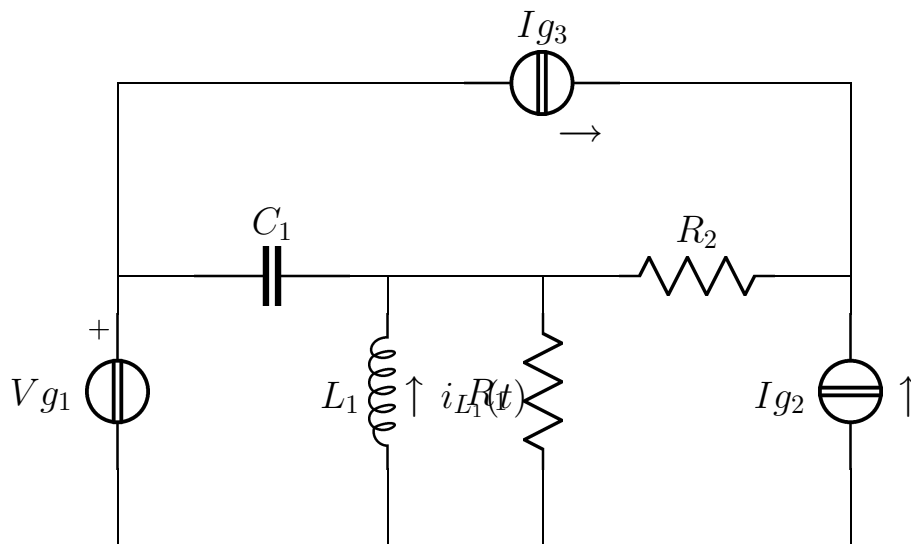


Esercizio ggcesame₂₀₁₆ – 01 – 29_{parallelo5}; Maglie

Risolvere il circuito in figura



$$\begin{aligned} \mathbf{V}_{g1} &= 2 - 2j \\ R_1 &= 1 \\ L_1 &= \frac{1}{2} \\ \mathbf{I}_{g2} &= -j \\ C_1 &= \frac{1}{2} \\ R_2 &= 1 \\ \mathbf{I}_{g3} &= -1 - j \\ \omega &= 2 \end{aligned}$$

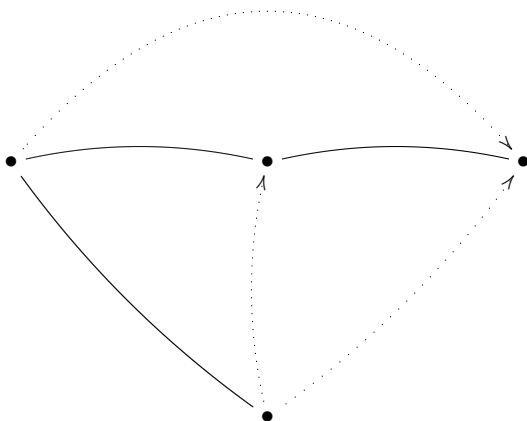
Semplificazioni serie/parallelo

$$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_1} = 1 - j$$

$$Z_a = \frac{1}{2} + \frac{1}{2}j$$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\left\{ \begin{array}{rclcl} (Z_a + \frac{1}{j\omega C_1})\mathbf{I}_1 & + \frac{1}{j\omega C_1}\mathbf{I}_2 & + \frac{1}{j\omega C_1}\mathbf{I}_3 & = & -\mathbf{V}_{\mathbf{g}1} \\ \frac{1}{j\omega C_1}\mathbf{I}_1 & + (\frac{1}{j\omega C_1} + R_2)\mathbf{I}_2 & + (\frac{1}{j\omega C_1} + R_2)\mathbf{I}_3 & = & -\mathbf{V}_{\mathbf{g}1} + \mathbf{V}_{\mathbf{x}2} \\ \frac{1}{j\omega C_1}\mathbf{I}_1 & + (\frac{1}{j\omega C_1} + R_2)\mathbf{I}_2 & + (\frac{1}{j\omega C_1} + R_2)\mathbf{I}_3 & = & \mathbf{V}_{\mathbf{x}3} \\ & \mathbf{I}_2 & & = & \mathbf{I}_{\mathbf{g}2} \\ & & \mathbf{I}_3 & = & \mathbf{I}_{\mathbf{g}3} \end{array} \right.$$

Sostituzione

$$\left\{ \begin{array}{rclcl} (\frac{1}{2} - \frac{1}{2}j)\mathbf{I}_1 & -j\mathbf{I}_2 & -j\mathbf{I}_3 & = & -2 + 2j \\ -j\mathbf{I}_1 & + (1-j)\mathbf{I}_2 & + (1-j)\mathbf{I}_3 & = & -2 + 2j + \mathbf{V}_{\mathbf{x}2} \\ -j\mathbf{I}_1 & + (1-j)\mathbf{I}_2 & + (1-j)\mathbf{I}_3 & = & \mathbf{V}_{\mathbf{x}3} \\ & \mathbf{I}_2 & & = & -j \\ & & \mathbf{I}_3 & = & -1 - j \end{array} \right.$$

Soluzione

$$\left\{ \begin{array}{rcl} \mathbf{I}_1 & = & -1 + j \\ \mathbf{I}_2 & = & -j \\ \mathbf{I}_3 & = & -1 - j \\ \mathbf{V}_{\mathbf{x}2} & = & -2j \\ \mathbf{V}_{\mathbf{x}3} & = & -2 \end{array} \right.$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{aligned} \mathbf{I}_{\mathbf{V}_{\mathbf{g}1}} &= -\mathbf{I}_1 - \mathbf{I}_2 = 1 & P_{c_{V_{g1}}} &= \frac{1}{2} \mathbf{V}_{\mathbf{g}1} \mathbf{I}_{\mathbf{V}_{\mathbf{g}1}}^* = 1 - j \\ \mathbf{V}_{\mathbf{I}_{\mathbf{g}2}} &= \mathbf{V}_{\mathbf{x}_{\mathbf{g}2}} = -2j & P_{c_{I_{g2}}} &= \frac{1}{2} \mathbf{V}_{\mathbf{I}_{\mathbf{g}2}} \mathbf{I}_{\mathbf{g}2}^* = 1 \\ \mathbf{V}_{\mathbf{I}_{\mathbf{g}3}} &= \mathbf{V}_{\mathbf{x}_{\mathbf{g}3}} = -2 & P_{c_{I_{g3}}} &= \frac{1}{2} \mathbf{V}_{\mathbf{I}_{\mathbf{g}3}} \mathbf{I}_{\mathbf{g}3}^* = 1 - j \end{aligned}$$

$$P_{c_{tot}} = 3 - 2j$$

Potenza attiva assorbita dai resistori:

$$\begin{aligned} \mathbf{I}_{\mathbf{R}_1} &= \frac{\mathbf{I}_1 Z_a}{R_1} = -1 & P_{a_{R_1}} &= \frac{1}{2} R_1 |\mathbf{I}_{\mathbf{R}_1}|^2 = \frac{1}{2} \\ \mathbf{I}_{\mathbf{R}_2} &= -\mathbf{I}_2 - \mathbf{I}_3 = 1 + 2j & P_{a_{R_2}} &= \frac{1}{2} R_2 |\mathbf{I}_{\mathbf{R}_2}|^2 = \frac{5}{2} \end{aligned}$$

$$P_{a_{tot}} = 3 = \Re\{P_{c_{tot}}\}$$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{aligned}\mathbf{V}_{\mathbf{C}_1} &= (-\mathbf{I}_1 - \mathbf{I}_2 - \mathbf{I}_3) \frac{1}{j\omega C_1} = 1 - 2j & Q_{C_1} &= -\frac{1}{2}\omega C_1 |\mathbf{V}_{\mathbf{C}_1}|^2 = -\frac{5}{2} \\ \mathbf{I}_{\mathbf{L}_1} &= \frac{\mathbf{I}_1 Z_a}{j\omega L_1} = j & Q_{L_1} &= \frac{1}{2}\omega L_1 |\mathbf{I}_{\mathbf{L}_1}|^2 = \frac{1}{2}\end{aligned}$$

$$Q_{tot} = -2 = \Im m\{P_{c_{tot}}\}$$

Calcolo tensioni e correnti

$$\mathbf{I}_{\mathbf{L}_1} = \frac{\mathbf{I}_1 Z_a}{j\omega L_1} = j$$

$$i_{L_1}(t) = \cos\left(2t + \frac{\pi}{2}\right)$$

Soluzioni:

$$\begin{aligned}V_{g_1} &= 2 - 2j; & I_{g_1} &= 1; & Pc_{V_{g_1}} &= 1 - j \\ V_{R_1} &= V_{L_1} = 1; & I_{R_1} + I_{L_1} &= -1 + j; & Pa_{R_1} &= \frac{1}{2} \\ Q_{L_1} &= \frac{1}{2} \\ V_{g_2} &= -2j; & I_{g_2} &= -j; & Pc_{I_{g_2}} &= 1 \\ V_{C_1} &= -1 + 2j; & I_{C_1} &= 2 + j; & Q_{C_1} &= -\frac{5}{2} \\ V_{R_2} &= -1 - 2j; & I_{R_2} &= 1 + 2j; & Pa_{R_2} &= \frac{5}{2} \\ V_{g_3} &= -2; & I_{g_3} &= -1 - j; & Pc_{I_{g_3}} &= 1 - j\end{aligned}$$