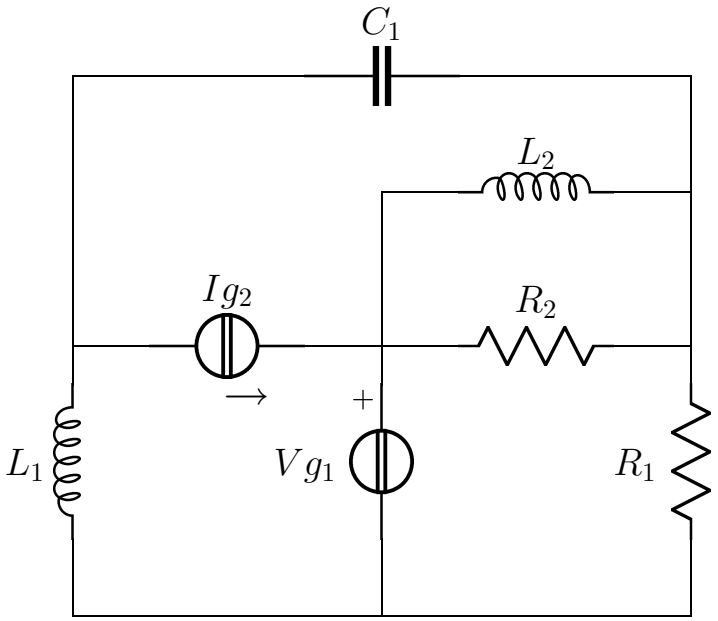
Esercizio ggcesame$_2$015 − 02 − 24$_B$4$_N$odi$_r$if2

Risolvere il circuito in figura



$L_1 = 1$
$\mathbf{V_{g_1}} = -j$
$R_1 = 1$
$\mathbf{I_{g_2}} = -1 - 3j$
$R_2 = 2$
$L_2 = \frac{2}{7}$
$C_1 = 2$
$\omega = 1$

Semplificazioni serie/parallelo
$Y_a = \frac{1}{R_2} + \frac{1}{j\omega L_2} = \frac{1}{2} - \frac{7}{2}j$
$Z_a = \frac{1}{25} + \frac{7}{25}j$

Risoluzione dell'esercizio con il metodo dei nodi
Sistema

$$\begin{cases} (\frac{1}{j\omega L_1} + j\omega C_1)\mathbf{E_1} & -j\omega C_1\mathbf{E_2} & -\frac{1}{j\omega L_1}\mathbf{E_3} &= -\mathbf{I_{g_2}} \\ -j\omega C_1\mathbf{E_1} & +(\frac{1}{R_1} + Y_a + j\omega C_1)\mathbf{E_2} & -\frac{1}{R_1}\mathbf{E_3} &= 0 \\ -\frac{1}{j\omega L_1}\mathbf{E_1} & -\frac{1}{R_1}\mathbf{E_2} & +(\frac{1}{j\omega L_1} + \frac{1}{R_1})\mathbf{E_3} &= -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} &= \mathbf{V_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} j\mathbf{E_1} & -2j\mathbf{E_2} & +j\mathbf{E_3} &= 1 + 3j \\ -2j\mathbf{E_1} & +(\frac{3}{2} - \frac{3}{2}j)\mathbf{E_2} & -\mathbf{E_3} &= 0 \\ j\mathbf{E_1} & -\mathbf{E_2} & +(1-j)\mathbf{E_3} &= -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} &= -j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} = 1 \\ \mathbf{E_2} = -1+j \\ \mathbf{E_3} = j \\ \mathbf{I_{x_1}} = -2-j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_1}} = -2-j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = \frac{1}{2}+j \\ \mathbf{V_{I_{g2}}} = -\mathbf{E_1} = -1 & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I}^*_{\mathbf{g2}} = \frac{1}{2}-\frac{3}{2}j \end{array}$$

$P_{c_{tot}} = 1 - \frac{1}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{R_1} = -1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}}{R_2} = -\frac{1}{2}+\frac{1}{2}j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = 1 = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{j\omega L_1} = -1-j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 1 \\ \mathbf{V_{C_1}} = \mathbf{E_2}-\mathbf{E_1} = -2+j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -5 \\ \mathbf{I_{L_2}} = \frac{\mathbf{E_2}}{j\omega L_2} = \frac{7}{2}+\frac{7}{2}j & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{7}{2} \end{array}$$

$Q_{tot} = -\frac{1}{2} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{L_1} = 1-j; & I_{L_1} = 1+j; & Q_{L_1} = 1 \\ V_{g_1} = -j; & I_{g_1} = -2-j; & Pc_{V_{g1}} = \frac{1}{2}+j \\ V_{R_1} = -1; & I_{R_1} = 1; & Pa_{R_1} = \frac{1}{2} \\ V_{g_2} = -1; & I_{g_2} = -1-3j; & Pc_{I_{g2}} = \frac{1}{2}-\frac{3}{2}j \\ V_{R_2} = V_{L_2} = -1+j; & I_{R_2}+I_{L_2} = -3-4j; & Pa_{R_2} = \frac{1}{2} \\ Q_{L_2} = \frac{7}{2} & & \\ V_{C_1} = -2+j; & I_{C_1} = 2+4j; & Q_{C_1} = -5 \end{array}$$