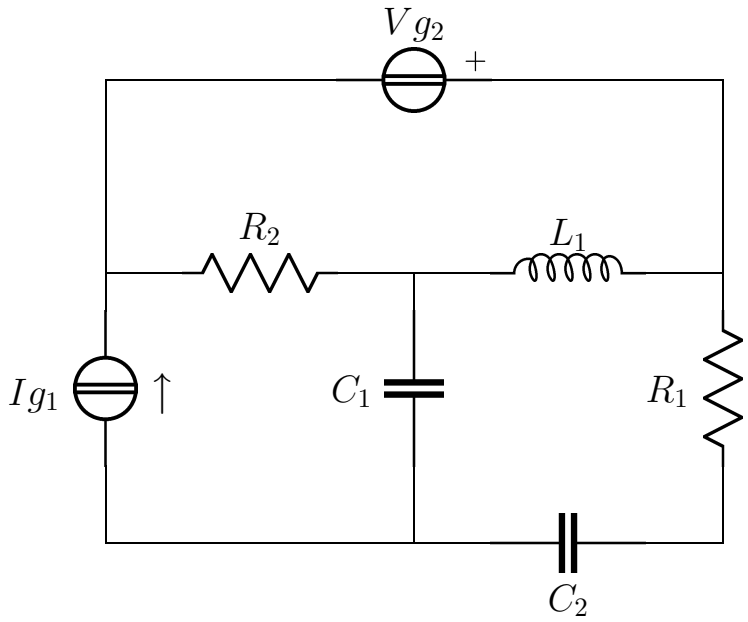
Esercizio ggcesame$_2$015 − 02 − 10$_A$2$_M$*aglie*

Risolvere il circuito in figura

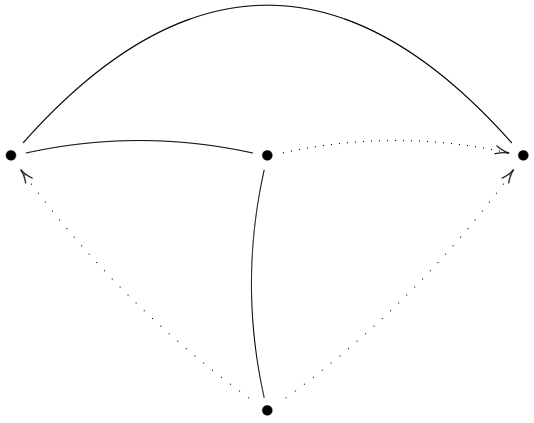


$\mathbf{I_{g_1}} = j$
$C_1 = 1$
$R_1 = \frac{5}{4}$
$C_2 = \frac{1}{5}$
$R_2 = 1$
$L_1 = 1$
$\mathbf{V_{g_2}} = 1 - 8j$
$\omega = 2$

Semplificazioni serie/parallelo
$Z_a = R_1 + \frac{1}{j\omega C_2} = \frac{5}{4} - \frac{5}{2}j$
$Y_a = \frac{4}{25} + \frac{8}{25}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (\frac{1}{j\omega C_1}+R_2)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+R_2)\mathbf{I_2} & +R_2\mathbf{I_3} & = & \mathbf{V_{x_1}} \\ (\frac{1}{j\omega C_1}+R_2)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+Z_a+R_2)\mathbf{I_2} & +R_2\mathbf{I_3} & = & -\mathbf{V_{g_2}} \\ R_2\mathbf{I_1} & +R_2\mathbf{I_2} & +(R_2+j\omega L_1)\mathbf{I_3} & = & -\mathbf{V_{g_2}} \\ \mathbf{I_1} & & & = & \mathbf{I_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} (1-\frac{1}{2}j)\mathbf{I_1} & +(1-\frac{1}{2}j)\mathbf{I_2} & +\mathbf{I_3} & = & \mathbf{V_{x_1}} \\ (1-\frac{1}{2}j)\mathbf{I_1} & +(\frac{9}{4}-3j)\mathbf{I_2} & +\mathbf{I_3} & = & -1+8j \\ \mathbf{I_1} & +\mathbf{I_2} & +(1+2j)\mathbf{I_3} & = & -1+8j \\ \mathbf{I_1} & & & = & j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & j \\ \mathbf{I_2} & = & -2 \\ \mathbf{I_3} & = & 3+j \\ \mathbf{V_{x_1}} & = & \frac{3}{2}+3j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = \mathbf{V_{x_1}} = \frac{3}{2}+3j & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{3}{2}-\frac{3}{4}j \\ \mathbf{I_{V_{g2}}} = -\mathbf{I_2}-\mathbf{I_3} = -1-j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = \frac{7}{2}+\frac{9}{2}j \end{array}$$

$P_{c_{tot}} = 5+\frac{15}{4}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_2} = -2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{5}{2} \\ \mathbf{I_{R_2}} = \mathbf{I_1}+\mathbf{I_2}+\mathbf{I_3} = 1+2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{5}{2} \end{array}$$

$P_{a_{tot}} = 5 = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = (-\mathbf{I_1}-\mathbf{I_2})\frac{1}{j\omega C_1} = -\frac{1}{2}-j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{5}{4} \\ \mathbf{I_{L_1}} = \mathbf{I_3} = 3+j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 10 \\ \mathbf{V_{C_2}} = \frac{\mathbf{I_2}}{j\omega C_2} = 5j & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -5 \end{array}$$

$Q_{tot} = \frac{15}{4} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{g_1} = \frac{3}{2} + 3j;$	$I_{g_1} = j;$	$Pc_{I_{g1}} = \frac{3}{2} - \frac{3}{4}j$
$V_{C_1} = \frac{1}{2} + j;$	$I_{C_1} = 2 - j;$	$Q_{C_1} = -\frac{5}{4}$
$V_{R_1} + V_{C_2} = \frac{5}{2} - 5j;$	$I_{R_1} = I_{C_2} = -2;$	$Pa_{R_1} = \frac{5}{2}$
$Q_{C_2} = -5$		
$V_{R_2} = -1 - 2j;$	$I_{R_2} = 1 + 2j;$	$Pa_{R_2} = \frac{5}{2}$
$V_{L_1} = 2 - 6j;$	$I_{L_1} = 3 + j;$	$Q_{L_1} = 10$
$V_{g_2} = 1 - 8j;$	$I_{g_2} = -1 - j;$	$Pc_{V_{g2}} = \frac{7}{2} + \frac{9}{2}j$