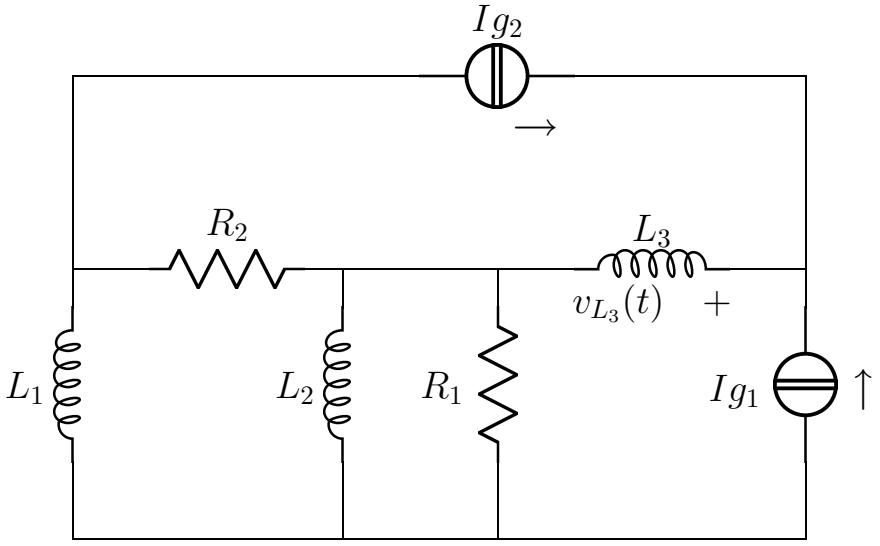
Esercizio ggcesame$_2$016 − 01 − 29$_P$arallelo4; NodiRif2

Risolvere il circuito in figura



$L_1 = 1$
$R_1 = 1$
$L_2 = \frac{1}{2}$
$\mathbf{I_{g_1}} = -1 + j$
$R_2 = 2$
$L_3 = 2$
$\mathbf{I_{g_2}} = -1 + j$
$\omega = 1$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_2} = 1 - 2j$
$Z_a = \frac{1}{5} + \frac{2}{5}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{j\omega L_1} + \frac{1}{R_2})\mathbf{E_1} & & -\frac{1}{j\omega L_1}\mathbf{E_3} & = & -\mathbf{I_{g_2}} \\ & \frac{1}{j\omega L_3}\mathbf{E_2} & & = & \mathbf{I_{g_1}} + \mathbf{I_{g_2}} \\ -\frac{1}{j\omega L_1}\mathbf{E_1} & & +(\frac{1}{j\omega L_1} + Y_a)\mathbf{E_3} & = & -\mathbf{I_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2} - j)\mathbf{E_1} & & +j\mathbf{E_3} & = & 1 - j \\ & -\frac{1}{2}j\mathbf{E_2} & & = & -2 + 2j \\ j\mathbf{E_1} & & +(1 - 3j)\mathbf{E_3} & = & 1 - j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} & = & 2 \\ \mathbf{E_2} & = & -4 - 4j \\ \mathbf{E_3} & = & 1 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = \mathbf{E_2} - \mathbf{E_3} = -5 - 4j & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{1}{2} + \frac{9}{2}j \\ \mathbf{V_{I_{g2}}} = \mathbf{E_2} - \mathbf{E_1} = -6 - 4j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = 1 + 5j \end{array}$$

$P_{c_{tot}} = \frac{3}{2} + \frac{19}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{-\mathbf{E_3}}{R_1} = -1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{-\mathbf{E_1}}{R_2} = -1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 1 \end{array}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{lll} \mathbf{I_{L_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{j\omega L_1} = -j & & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{L_3}} = \frac{\mathbf{E_2}}{j\omega L_3} = -2 + 2j & & Q_{L_3} = \frac{1}{2}\omega L_3|\mathbf{I_{L_3}}|^2 = 8 \\ \mathbf{I_{L_2}} = \frac{-\mathbf{E_3}}{j\omega L_2} = 2j & & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = 1 \end{array}$$

$Q_{tot} = \frac{19}{2} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{V_{L_3}} = \mathbf{E_2} = -4 - 4j$$

$$v_{L_3}(t) = 4\sqrt{2}\cos\left(t + \frac{5\pi}{4}\right)$$

Soluzioni:

$$\begin{array}{lll} V_{L_1} = 1; & I_{L_1} = j; & Q_{L_1} = \frac{1}{2} \\ V_{R_1} = V_{L_2} = -1; & I_{R_1} + I_{L_2} = 1 - 2j; & Pa_{R_1} = \frac{1}{2} \\ Q_{L_2} = 1 & & \\ V_{g_1} = -5 - 4j; & I_{g_1} = -1 + j; & Pc_{I_{g1}} = \frac{1}{2} + \frac{9}{2}j \\ V_{R_2} = -2; & I_{R_2} = 1; & Pa_{R_2} = 1 \\ V_{L_3} = -4 - 4j; & I_{L_3} = 2 - 2j; & Q_{L_3} = 8 \\ V_{g_2} = -6 - 4j; & I_{g_2} = -1 + j; & Pc_{I_{g2}} = 1 + 5j \end{array}$$