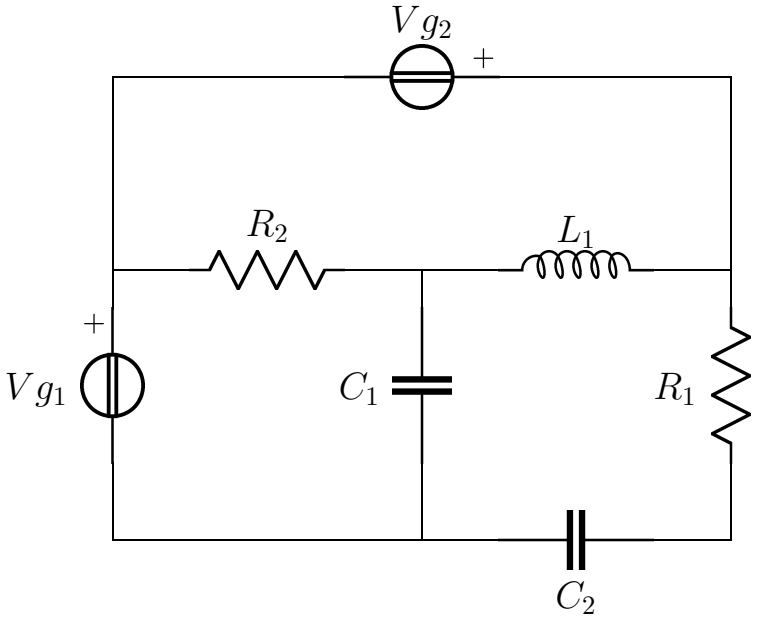
Esercizio ggcesame$_2$015 − 02 − 10$_A$5$_N$odi$_r$if1

Risolvere il circuito in figura



$\mathbf{V_{g_1}} = 3 + 4j$
$C_1 = 1$
$R_1 = \frac{1}{2}$
$C_2 = \frac{1}{2}$
$R_2 = 2$
$L_1 = 2$
$\mathbf{V_{g_2}} = -2 - 8j$
$\omega = 1$

Semplificazioni serie/parallelo
$Z_a = R_1 + \frac{1}{j\omega C_2} = \frac{1}{2} - 2j$
$Y_a = \frac{2}{17} + \frac{8}{17}j$

Risoluzione dell'esercizio con il metodo dei nodi
Sistema

$$\begin{cases} (j\omega C_1 + \frac{1}{R_2} + \frac{1}{j\omega L_1})\mathbf{E_1} & -\frac{1}{j\omega L_1}\mathbf{E_2} & -j\omega C_1\mathbf{E_3} &= 0 \\ -\frac{1}{j\omega L_1}\mathbf{E_1} & +(Y_a + \frac{1}{j\omega L_1})\mathbf{E_2} & -Y_a\mathbf{E_3} &= \mathbf{I_{x_2}} \\ -j\omega C_1\mathbf{E_1} & -Y_a\mathbf{E_2} & +(j\omega C_1 + Y_a)\mathbf{E_3} &= -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} &= \mathbf{V_{g_1}} \\ & \mathbf{E_2} & &= \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2} + \frac{1}{2}j)\mathbf{E_1} & +\frac{1}{2}j\mathbf{E_2} & -j\mathbf{E_3} &= 0 \\ \frac{1}{2}j\mathbf{E_1} & +(\frac{2}{17} - \frac{1}{34}j)\mathbf{E_2} & +(-\frac{2}{17} - \frac{8}{17}j)\mathbf{E_3} &= \mathbf{I_{x_2}} \\ -j\mathbf{E_1} & +(-\frac{2}{17} - \frac{8}{17}j)\mathbf{E_2} & +(\frac{2}{17} + \frac{25}{17}j)\mathbf{E_3} &= -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} &= 3 + 4j \\ & \mathbf{E_2} & &= -2 - 8j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} &= -2-2j \\ \mathbf{E_2} &= -2-8j \\ \mathbf{E_3} &= -3-4j \\ \mathbf{I_{x_1}} &= j \\ \mathbf{I_{x_2}} &= -1 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_1}} = j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = 2 - \frac{3}{2}j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_1}} = -1 & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I}^*_{\mathbf{V_{g2}}} = 1 + 4j \end{array}$$

$P_{c_{tot}} = 3 + \frac{5}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{Z_a} = 2 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_1}}{R_2} = -1 - j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 2 \end{array}$$

$P_{a_{tot}} = 3 = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{lll} \mathbf{V_{C_1}} = \mathbf{E_1} - \mathbf{E_3} = 1 + 2j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{5}{2} \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{j\omega L_1} = -3 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 9 \\ \mathbf{V_{C_2}} = \frac{(\mathbf{E_2}-\mathbf{E_3})Y_a}{j\omega C_2} = -4j & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -4 \end{array}$$

$Q_{tot} = \frac{5}{2} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{g_1} = 3 + 4j; & I_{g_1} = j; & Pc_{V_{g1}} = 2 - \frac{3}{2}j \\ V_{C_1} = 1 + 2j; & I_{C_1} = 2 - j; & Q_{C_1} = -\frac{5}{2} \\ V_{R_1} + V_{C_2} = 1 - 4j; & I_{R_1} = I_{C_2} = -2; & Pa_{R_1} = 1 \\ Q_{C_2} = -4 & & \\ V_{R_2} = -2 - 2j; & I_{R_2} = 1 + j; & Pa_{R_2} = 2 \\ V_{L_1} = -6j; & I_{L_1} = 3; & Q_{L_1} = 9 \\ V_{g_2} = -2 - 8j; & I_{g_2} = -1; & Pc_{V_{g2}} = 1 + 4j \end{array}$$