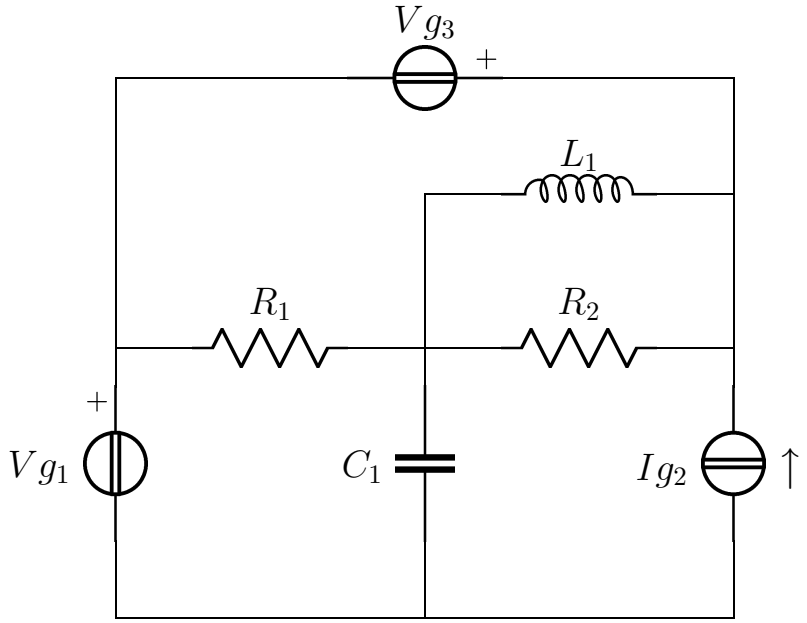
Esercizio ggcesame$_2$015 − 02 − 24$_B$3$_M$*aglie*

Risolvere il circuito in figura

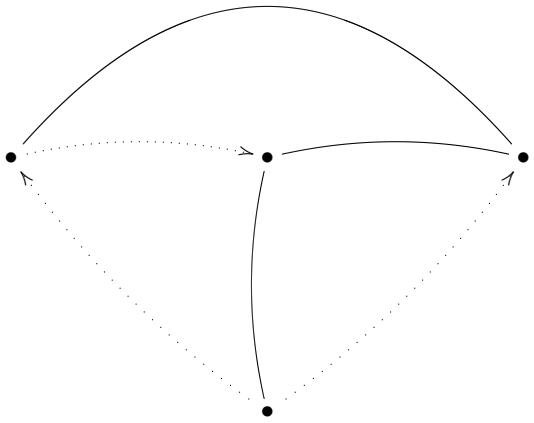


$\mathbf{V_{g_1}} = -2$
$C_1 = 1$
$\mathbf{I_{g_2}} = -1$
$R_1 = 2$
$R_2 = 1$
$L_1 = 1$
$\mathbf{V_{g_3}} = 1 + 2j$
$\omega = 1$

Semplificazioni serie/parallelo
$Y_a = \frac{1}{R_2} + \frac{1}{j\omega L_1} = 1 - j$
$Z_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (\frac{1}{j\omega C_1}+Z_a)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+Z_a)\mathbf{I_2} & -Z_a\mathbf{I_3} & = & \mathbf{V_{g_1}}+\mathbf{V_{g_3}} \\ (\frac{1}{j\omega C_1}+Z_a)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+Z_a)\mathbf{I_2} & -Z_a\mathbf{I_3} & = & \mathbf{V_{x_2}} \\ -Z_a\mathbf{I_1} & -Z_a\mathbf{I_2} & +(R_1+Z_a)\mathbf{I_3} & = & -\mathbf{V_{g_3}} \\ & \mathbf{I_2} & & = & \mathbf{I_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2}-\frac{1}{2}j)\mathbf{I_1} & +(\frac{1}{2}-\frac{1}{2}j)\mathbf{I_2} & +(-\frac{1}{2}-\frac{1}{2}j)\mathbf{I_3} & = & -1+2j \\ (\frac{1}{2}-\frac{1}{2}j)\mathbf{I_1} & +(\frac{1}{2}-\frac{1}{2}j)\mathbf{I_2} & +(-\frac{1}{2}-\frac{1}{2}j)\mathbf{I_3} & = & \mathbf{V_{x_2}} \\ (-\frac{1}{2}-\frac{1}{2}j)\mathbf{I_1} & +(-\frac{1}{2}-\frac{1}{2}j)\mathbf{I_2} & +(\frac{5}{2}+\frac{1}{2}j)\mathbf{I_3} & = & -1-2j \\ & \mathbf{I_2} & & = & -1 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & -1 \\ \mathbf{I_2} & = & -1 \\ \mathbf{I_3} & = & -1-j \\ \mathbf{V_{x_2}} & = & -1+2j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_1} = -1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = 1 \\ \mathbf{V_{I_{g2}}} = \mathbf{V_{x_g 2}} = -1+2j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = \frac{1}{2}-j \\ \mathbf{I_{V_{g3}}} = \mathbf{I_1}-\mathbf{I_3} = j & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I^*_{V_{g3}}} = 1-\frac{1}{2}j \end{array}$$

$P_{c_{tot}} = \frac{5}{2}-\frac{3}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_3} = -1-j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 2 \\ \mathbf{I_{R_2}} = \frac{(-\mathbf{I_1}-\mathbf{I_2}+\mathbf{I_3})Z_a}{R_2} = 1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{5}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = (-\mathbf{I_1} - \mathbf{I_2})\frac{1}{j\omega C_1} = -2j & Q_{C_1} = -\frac{1}{2}\omega C_1 |\mathbf{V_{C_1}}|^2 = -2 \\ \mathbf{I_{L_1}} = \frac{(-\mathbf{I_1} - \mathbf{I_2} + \mathbf{I_3})Z_a}{j\omega L_1} = -j & Q_{L_1} = \frac{1}{2}\omega L_1 |\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -\frac{3}{2} = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{g_1} = -2; & I_{g_1} = -1; & Pc_{V_{g1}} = 1 \\ V_{C_1} = 2j; & I_{C_1} = 2; & Q_{C_1} = -2 \\ V_{g_2} = -1 + 2j; & I_{g_2} = -1; & Pc_{I_{g2}} = \frac{1}{2} - j \\ V_{R_1} = 2 + 2j; & I_{R_1} = -1 - j; & Pa_{R_1} = 2 \\ V_{R_2} = V_{L_1} = -1; & I_{R_2} + I_{L_1} = 1 - j; & Pa_{R_2} = \frac{1}{2} \\ Q_{L_1} = \frac{1}{2} & & \\ V_{g_3} = 1 + 2j; & I_{g_3} = j; & Pc_{V_{g3}} = 1 - \frac{1}{2}j \end{array}$$