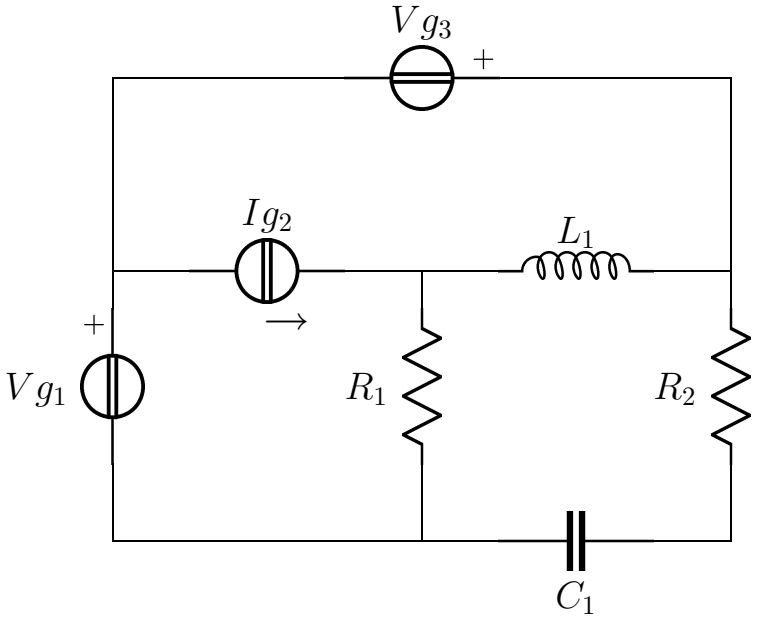
Esercizio A1rif1

Risolvere il circuito in figura



$\mathbf{V_{g_1}} = \frac{3}{2} - 2j$
$R_1 = 2$
$R_2 = \frac{1}{2}$
$C_1 = \frac{1}{5}$
$\mathbf{I_{g_2}} = 1$
$L_1 = \frac{1}{2}$
$\mathbf{V_{g_3}} = \frac{3}{2}$
$\omega = 2$

Semplificazioni serie/parallelo

$Z_a = R_2 + \frac{1}{j\omega C_1} = \frac{1}{2} - \frac{5}{2}j$
$Y_a = \frac{1}{13} + \frac{5}{13}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{R_1} + \frac{1}{j\omega L_1})\mathbf{E_1} & -\frac{1}{j\omega L_1}\mathbf{E_2} & -\frac{1}{R_1}\mathbf{E_3} & = & \mathbf{I_{g_2}} \\ -\frac{1}{j\omega L_1}\mathbf{E_1} & +(Y_a + \frac{1}{j\omega L_1})\mathbf{E_2} & -Y_a\mathbf{E_3} & = & \mathbf{I_{x_3}} \\ -\frac{1}{R_1}\mathbf{E_1} & -Y_a\mathbf{E_2} & +(\frac{1}{R_1} + Y_a)\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_3}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{1}{2}-j)\mathbf{E_1} & +j\mathbf{E_2} & -\frac{1}{2}\mathbf{E_3} & = & 1 \\ j\mathbf{E_1} & +(\frac{1}{13}-\frac{8}{13}j)\mathbf{E_2} & +(-\frac{1}{13}-\frac{5}{13}j)\mathbf{E_3} & = & \mathbf{I_{x_3}} \\ -\frac{1}{2}\mathbf{E_1} & +(-\frac{1}{13}-\frac{5}{13}j)\mathbf{E_2} & +(\frac{15}{26}+\frac{5}{13}j)\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \frac{3}{2}-2j \\ & \mathbf{E_2} & & = & \frac{3}{2} \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} & = & \frac{1}{2} \\ \mathbf{E_2} & = & \frac{3}{2} \\ \mathbf{E_3} & = & -\frac{3}{2}+2j \\ \mathbf{I_{x_1}} & = & 2 \\ \mathbf{I_{x_3}} & = & 1 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_2}} = 2 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = \frac{3}{2}-2j \\ \mathbf{V_{I_{g2}}} = \mathbf{E_1} = \frac{1}{2} & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = \frac{1}{4} \\ \mathbf{I_{V_{g3}}} = \mathbf{I_{x_2}} = 1 & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I^*_{V_{g3}}} = \frac{3}{4} \end{array}$$

$P_{c_{tot}} = \frac{5}{2}-2j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{R_1} = 1-j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 2 \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{Z_a} = 1+j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{5}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_1}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{j\omega L_1} = -j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \\ \mathbf{V_{C_1}} = \frac{(\mathbf{E_2}-\mathbf{E_3})Y_a}{j\omega C_1} = \frac{5}{2}-\frac{5}{2}j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{5}{2} \end{array}$$

$Q_{tot} = -2 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{g_1} = \frac{3}{2} - 2j;$	$I_{g_1} = 2;$	$Pc_{V_{g1}} = \frac{3}{2} - 2j$
$V_{R_1} = 2 - 2j;$	$I_{R_1} = -1 + j;$	$Pa_{R_1} = 2$
$V_{R_2} + V_{C_1} = 3 - 2j;$	$I_{R_2} = I_{C_1} = -1 - j;$	$Pa_{R_2} = \frac{1}{2}$
$Q_{C_1} = -\frac{5}{2}$		
$V_{g_2} = \frac{1}{2};$	$I_{g_2} = 1;$	$Pc_{I_{g2}} = \frac{1}{4}$
$V_{L_1} = 1;$	$I_{L_1} = j;$	$Q_{L_1} = \frac{1}{2}$
$V_{g_3} = \frac{3}{2};$	$I_{g_3} = 1;$	$Pc_{V_{g3}} = \frac{3}{4}$