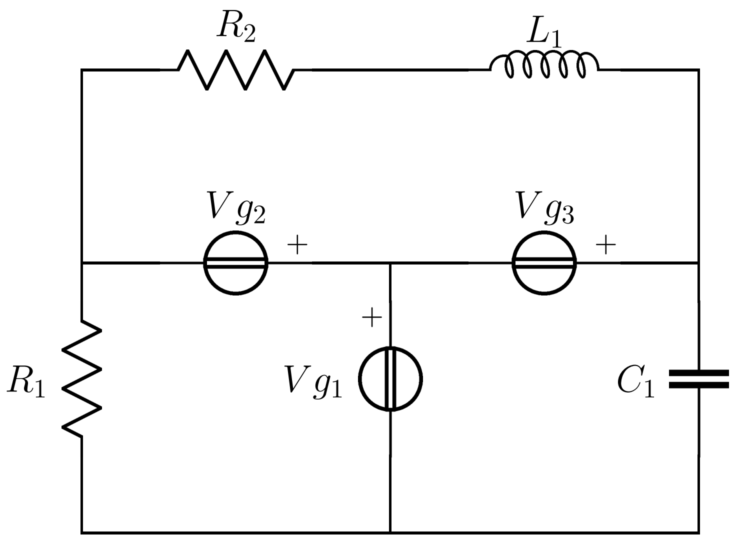
Esercizio d1 risolto 1

Risolvere il circuito in figura



$R_1 = 1$
$v_{g_1}(t) = \cos(t + \pi)$
$C_1 = 1$
$v_{g_2}(t) = \cos(t + \frac{\pi}{2})$
$v_{g_3}(t) = 2\cos(t)$
$R_2 = 2$
$L_1 = 1$

Fasori

$\mathbf{V_{g_3}} = 2$
$\mathbf{V_{g_2}} = j$
$\mathbf{V_{g_1}} = -1$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_1 = 2 + j$
$Y_a = \frac{2}{5} - \frac{1}{5}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{R_1} + Y_a)\mathbf{E_1} & -Y_a\mathbf{E_2} & -\frac{1}{R_1}\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ -Y_a\mathbf{E_1} & +(j\omega C_1 + Y_a)\mathbf{E_2} & -j\omega C_1\mathbf{E_3} & = & \mathbf{I_{x_3}} \\ -\frac{1}{R_1}\mathbf{E_1} & -j\omega C_1\mathbf{E_2} & +(\frac{1}{R_1} + j\omega C_1)\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_2}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_3}} \end{cases}$$

Sostituzione

$$\left\{\begin{array}{rrrcr}
(\frac{7}{5}-\frac{1}{5}j)\mathbf{E_1} & +(-\frac{2}{5}+\frac{1}{5}j)\mathbf{E_2} & -\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\
(-\frac{2}{5}+\frac{1}{5}j)\mathbf{E_1} & +(\frac{2}{5}+\frac{4}{5}j)\mathbf{E_2} & -j\mathbf{E_3} & = & \mathbf{I_{x_3}} \\
-\mathbf{E_1} & -j\mathbf{E_2} & +(1+j)\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\
 & & -\mathbf{E_3} & = & -1 \\
-\mathbf{E_1} & & & = & j \\
 & \mathbf{E_2} & & = & 2
\end{array}\right.$$

Soluzione

$$\left\{\begin{array}{rcr}
\mathbf{E_1} & = & -j \\
\mathbf{E_2} & = & 2 \\
\mathbf{E_3} & = & 1 \\
\mathbf{I_{x_1}} & = & -1 \\
\mathbf{I_{x_2}} & = & 2+j \\
\mathbf{I_{x_3}} & = & 1+j
\end{array}\right.$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll}
\mathbf{I_{V_{g1}}} = \mathbf{I_{x_1}} = -1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = \frac{1}{2} \\
\mathbf{I_{V_{g2}}} = \mathbf{I_{x_1}} = 2+j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = \frac{1}{2}+j \\
\mathbf{I_{V_{g3}}} = \mathbf{I_{x_1}} = 1+j & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I^*_{V_{g3}}} = 1-j
\end{array}$$

$P_{c_{tot}} = 2$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll}
\mathbf{I_{R_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{R_1} = -1-j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\
\mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{Z_a} = 1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 1
\end{array}$$

$P_{a_{tot}} = 2 = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll}
\mathbf{V_{C_1}} = \mathbf{E_2} - \mathbf{E_3} = 1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{1}{2} \\
\mathbf{I_{L_1}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{Z_a} = 1 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2}
\end{array}$$

$Q_{tot} = 0 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{R_1} = -1 - j;$	$I_{R_1} = 1 + j;$	$Pa_{R_1} = 1$
$V_{g_1} = -1;$	$I_{g_1} = -1;$	$Pc_{V_{g1}} = \frac{1}{2}$
$V_{C_1} = 1;$	$I_{C_1} = -j;$	$Q_{C_1} = -\frac{1}{2}$
$V_{g_2} = j;$	$I_{g_2} = 2 + j;$	$Pc_{V_{g2}} = \frac{1}{2} + j$
$V_{g_3} = 2;$	$I_{g_3} = 1 + j;$	$Pc_{V_{g3}} = 1 - j$
$V_{R_2} + V_{L_1} = 2 + j;$	$I_{R_2} = I_{L_1} = -1;$	$Pa_{R_2} = 1$
$Q_{L_1} = \frac{1}{2}$		