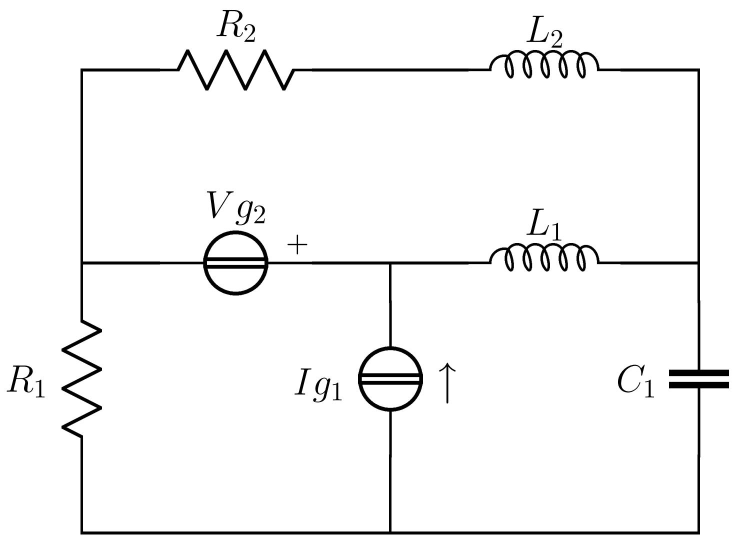
Esercizio b1 risolto 3

Risolvere il circuito in figura



$R_1 = 1$
$i_{g_1}(t) = \sqrt{10}\cos(t - \arctan(\frac{1}{3}))$
$C_1 = 1$
$v_{g_2}(t) = \cos(t + \pi)$
$L_1 = 1$
$R_2 = 1$
$L_2 = 1$

Fasori

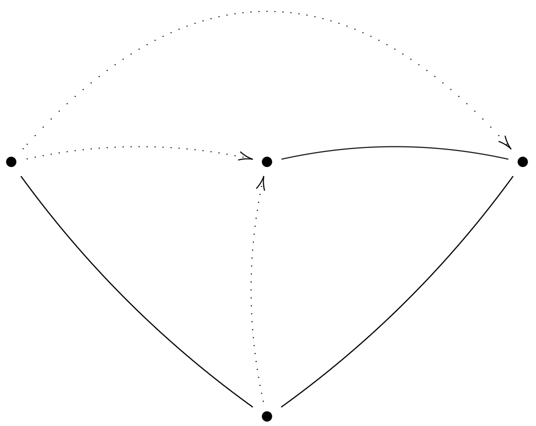
$\mathbf{V_{g_2}} = -1$
$\mathbf{I_{g_1}} = 3 - j$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_2 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases}
(\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_1} & +(\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_2} & +\frac{1}{j\omega C_1}\mathbf{I_3} & = & \mathbf{V_{x_1}} \\
(\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_1} & +(R_1+\frac{1}{j\omega C_1}+j\omega L_1)\mathbf{I_2} & +(R_1+\frac{1}{j\omega C_1})\mathbf{I_3} & = & \mathbf{V_{g_2}} \\
\frac{1}{j\omega C_1}\mathbf{I_1} & +(R_1+\frac{1}{j\omega C_1})\mathbf{I_2} & +(R_1+\frac{1}{j\omega C_1}+Z_a)\mathbf{I_3} & = & 0 \\
\mathbf{I_1} & & & = & \mathbf{I_{g_1}}
\end{cases}$$

Sostituzione

$$\begin{cases}
 & & -j\mathbf{I_3} & = & \mathbf{V_{x_1}} \\
 & \mathbf{I_2} & +(1-j)\mathbf{I_3} & = & -1 \\
-j\mathbf{I_1} & +(1-j)\mathbf{I_2} & +2\mathbf{I_3} & = & 0 \\
\mathbf{I_1} & & & = & 3-j
\end{cases}$$

Soluzione

$$\begin{cases}
\mathbf{I_1} & = & 3-j \\
\mathbf{I_2} & = & -2+j \\
\mathbf{I_3} & = & 1 \\
\mathbf{V_{x_1}} & = & -j
\end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll}
\mathbf{V_{I_{g1}}} = \mathbf{V_{x_1}} = -j & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{1}{2} - \frac{3}{2}j \\
\mathbf{I_{V_{g2}}} = \mathbf{I_2} = -2+j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 1 + \frac{1}{2}j
\end{array}$$

$P_{c_{tot}} = \frac{3}{2} - j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_2} + \mathbf{I_3} = -1 + j & P_{a_{R_1}} = \frac{1}{2} R_1 |\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \mathbf{I_3} = 1 & P_{a_{R_2}} = \frac{1}{2} R_2 |\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = (-\mathbf{I_1} - \mathbf{I_2} - \mathbf{I_3}) \frac{1}{j\omega C_1} = 2j & Q_{C_1} = -\frac{1}{2} \omega C_1 |\mathbf{V_{C_1}}|^2 = -2 \\ \mathbf{I_{L_1}} = \mathbf{I_1} + \mathbf{I_2} = 1 & Q_{L_1} = \frac{1}{2} \omega L_1 |\mathbf{I_{L_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{L_2}} = \mathbf{I_3} = 1 & Q_{L_2} = \frac{1}{2} \omega L_2 |\mathbf{I_{L_2}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -1 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$$\begin{array}{lll} V_{R_1} = 1 - j; & I_{R_1} = -1 + j; & Pa_{R_1} = 1 \\ V_{g_1} = -j; & I_{g_1} = 3 - j; & Pc_{I_{g1}} = \frac{1}{2} - \frac{3}{2} j \\ V_{C_1} = -2j; & I_{C_1} = -2; & Q_{C_1} = -2 \\ V_{g_2} = -1; & I_{g_2} = -2 + j; & Pc_{V_{g2}} = 1 + \frac{1}{2} j \\ V_{L_1} = -j; & I_{L_1} = 1; & Q_{L_1} = \frac{1}{2} \\ V_{R_2} + V_{L_2} = -1 - j; & I_{R_2} = I_{L_2} = 1; & Pa_{R_2} = \frac{1}{2} \\ Q_{L_2} = \frac{1}{2} & & \end{array}$$