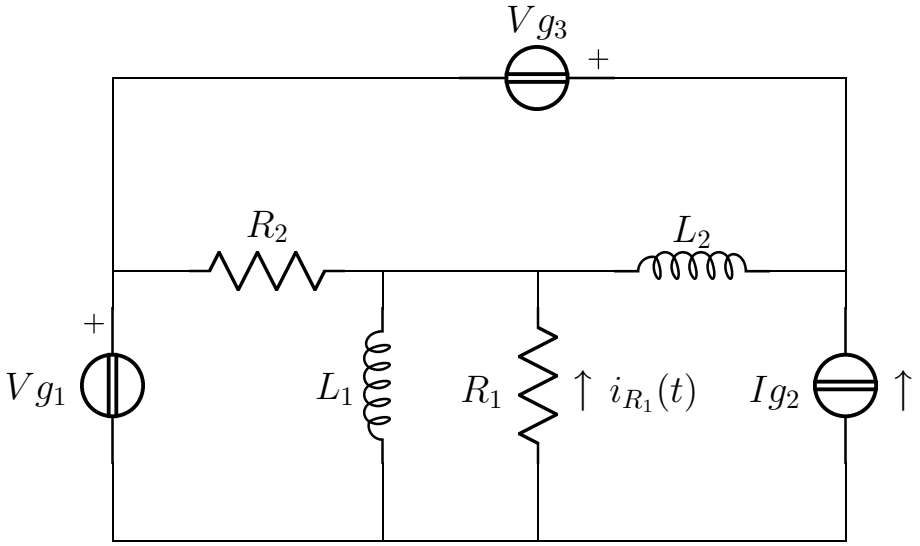
Esercizio B2 risolto

Risolvere il circuito in figura



$\mathbf{V_{g_1}} = -5 + 2j$
$R_1 = \frac{1}{3}$
$L_1 = 1$
$\mathbf{I_{g_2}} = -2$
$R_2 = 2$
$L_2 = 1$
$\mathbf{V_{g_3}} = 4 - 3j$
$\omega = 1$

Semplificazioni serie/parallelo
$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_1} = 3 - j$
$Z_a = \frac{3}{10} + \frac{1}{10}j$

Risoluzione dell'esercizio con il metodo dei nodi
Sistema

$$\begin{cases} (Y_a + \frac{1}{R_2} + \frac{1}{j\omega L_2})\mathbf{E_1} & -\frac{1}{j\omega L_2}\mathbf{E_2} & -Y_a\mathbf{E_3} & = & 0 \\ -\frac{1}{j\omega L_2}\mathbf{E_1} & +\frac{1}{j\omega L_2}\mathbf{E_2} & & = & \mathbf{I_{g_2}} + \mathbf{I_{x_3}} \\ -Y_a\mathbf{E_1} & & +Y_a\mathbf{E_3} & = & -\mathbf{I_{g_2}} - \mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_3}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{7}{2} - 2j)\mathbf{E_1} & +j\mathbf{E_2} & +(-3+j)\mathbf{E_3} & = & 0 \\ j\mathbf{E_1} & -j\mathbf{E_2} & & = & -2 + \mathbf{I_{x_3}} \\ (-3+j)\mathbf{E_1} & & +(3-j)\mathbf{E_3} & = & 2 - \mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & -5 + 2j \\ & \mathbf{E_2} & & = & 4 - 3j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} &= 4-2j \\ \mathbf{E_2} &= 4-3j \\ \mathbf{E_3} &= 5-2j \\ \mathbf{I_{x_1}} &= -1+j \\ \mathbf{I_{x_3}} &= 1 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_2}} = -1+j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = \frac{7}{2}+\frac{3}{2}j \\ \mathbf{V_{I_{g2}}} = \mathbf{E_2}-\mathbf{E_3} = -1-j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I}^*_{\mathbf{g2}} = 1+j \\ \mathbf{I_{V_{g3}}} = \mathbf{I_{x_2}} = 1 & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I}^*_{\mathbf{V_{g3}}} = 2-\frac{3}{2}j \end{array}$$

$P_{c_{tot}} = \frac{13}{2}+j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_3}-\mathbf{E_1}}{R_1} = 3 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{3}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_1}}{R_2} = 2-j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 5 \end{array}$$

$P_{a_{tot}} = \frac{13}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{j\omega L_2} = -1 & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{1}{2} \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{j\omega L_1} = j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = 1 = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{R_1}} = \frac{\mathbf{E_3}-\mathbf{E_1}}{R_1} = 3$$

$i_{R_1}(t) = 3\cos(t)$

Soluzioni:

$V_{g_1} = -5 + 2j;$	$I_{g_1} = -1 + j;$	$Pc_{V_{g1}} = \frac{7}{2} + \frac{3}{2}j$
$V_{R_1} = V_{L_1} = -1;$	$I_{R_1} + I_{L_1} = 3 - j;$	$Pa_{R_1} = \frac{3}{2}$
$Q_{L_1} = \frac{1}{2}$		
$V_{g_2} = -1 - j;$	$I_{g_2} = -2;$	$Pc_{I_{g2}} = 1 + j$
$V_{R_2} = 4 - 2j;$	$I_{R_2} = -2 + j;$	$Pa_{R_2} = 5$
$V_{L_2} = -j;$	$I_{L_2} = 1;$	$Q_{L_2} = \frac{1}{2}$
$V_{g_3} = 4 - 3j;$	$I_{g_3} = 1;$	$Pc_{V_{g3}} = 2 - \frac{3}{2}j$