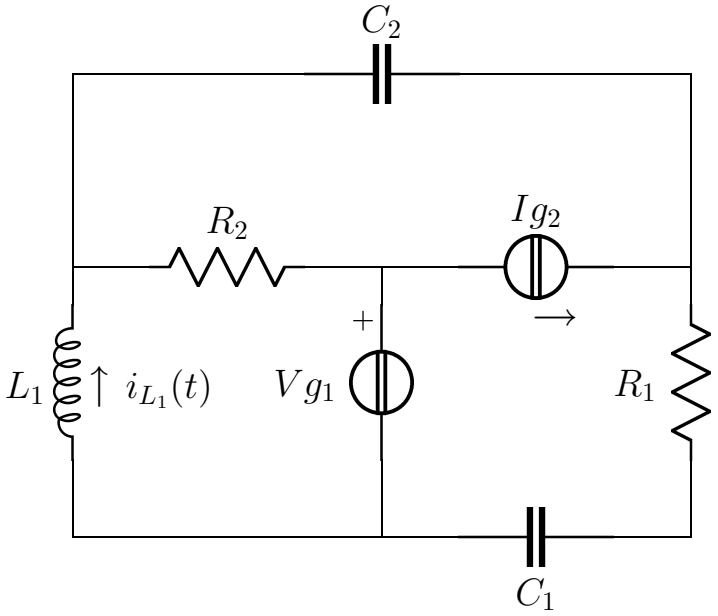
Esercizio A risolto

Risolvere il circuito in figura



$L_1 = 1$
$v_{g_1}(t) = \cos(t + \pi)$
$R_1 = 1$
$C_1 = 1$
$R_2 = \frac{1}{2}$
$i_{g_2}(t) = \sqrt{13}\cos(t - \arctan(\frac{2}{3}) + \pi)$
$C_2 = 2$

Fasori

$\mathbf{I_{g_2}} = -3 + 2j$
$\mathbf{V_{g_1}} = -1$

Semplificazioni serie/parallelo

$Z_a = R_1 + \frac{1}{j\omega C_1} = 1 - j$
$Y_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{j\omega L_1} + \frac{1}{R_2} + j\omega C_2)\mathbf{E_1} & -\frac{1}{R_2}\mathbf{E_2} & -j\omega C_2\mathbf{E_3} & = & 0 \\ -\frac{1}{R_2}\mathbf{E_1} & +\frac{1}{R_2}\mathbf{E_2} & & = & -\mathbf{I_{g_2}} + \mathbf{I_{x_1}} \\ -j\omega C_2\mathbf{E_1} & & +(Y_a + j\omega C_2)\mathbf{E_3} & = & \mathbf{I_{g_2}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} (2+j)\mathbf{E_1} & -2\mathbf{E_2} & -2j\mathbf{E_3} & = & 0 \\ -2\mathbf{E_1} & +2\mathbf{E_2} & & = & 3-2j+\mathbf{I_{x_1}} \\ -2j\mathbf{E_1} & & +(\frac{1}{2}+\frac{5}{2}j)\mathbf{E_3} & = & -3+2j \\ & \mathbf{E_2} & & = & -1 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} & = & -2 \\ \mathbf{E_2} & = & -1 \\ \mathbf{E_3} & = & -1+j \\ \mathbf{I_{x_1}} & = & -1+2j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_1}} = -1+2j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = \frac{1}{2}+j \\ \mathbf{V_{I_{g2}}} = \mathbf{E_3}-\mathbf{E_2} = j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = 1-\frac{3}{2}j \end{array}$$

$P_{c_{tot}} = \frac{3}{2} - \frac{1}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_3}}{Z_a} = -1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{R_2} = 2 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 1 \end{array}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_1}} = \frac{-\mathbf{E_1}}{j\omega L_1} = -2j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 2 \\ \mathbf{V_{C_2}} = \mathbf{E_3}-\mathbf{E_1} = 1+j & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -2 \\ \mathbf{V_{C_1}} = \frac{\mathbf{E_3}Y_a}{j\omega C_1} = j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{1}{2} \end{array}$$

$Q_{tot} = -\frac{1}{2} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{L_1}} = \frac{-\mathbf{E_1}}{j\omega L_1} = -2j$$

$i_{L_1}(t) = 2\cos(t - \frac{\pi}{2})$

Soluzioni:

$V_{L_1} = -2;$	$I_{L_1} = -2j;$	$Q_{L_1} = 2$
$V_{g_1} = -1;$	$I_{g_1} = -1 + 2j;$	$Pc_{V_{g1}} = \frac{1}{2} + j$
$V_{R_1} + V_{C_1} = -1 + j;$	$I_{R_1} = I_{C_1} = 1;$	$Pa_{R_1} = \frac{1}{2}$
$Q_{C_1} = -\frac{1}{2}$		
$V_{R_2} = 1;$	$I_{R_2} = -2;$	$Pa_{R_2} = 1$
$V_{g_2} = j;$	$I_{g_2} = -3 + 2j;$	$Pc_{I_{g2}} = 1 - \frac{3}{2}j$
$V_{C_2} = 1 + j;$	$I_{C_2} = 2 - 2j;$	$Q_{C_2} = -2$