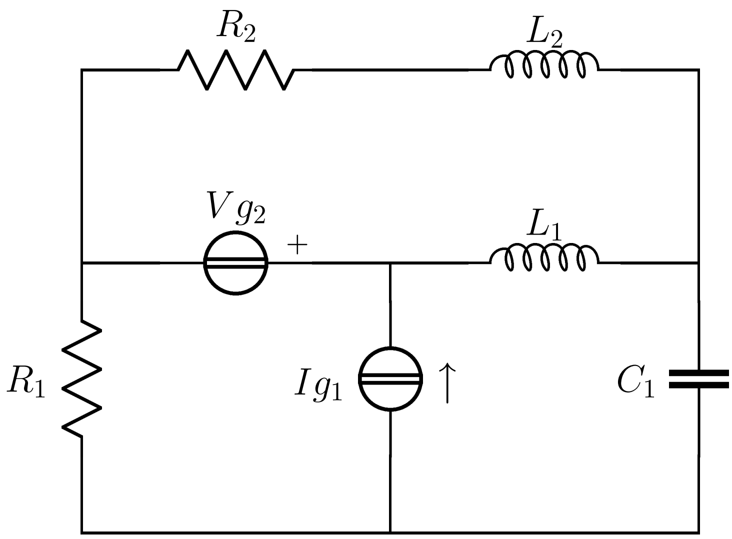
Esercizio B1 risolto 2

Risolvere il circuito in figura



$R_1 = 1$
$i_{g_1}(t) = \sqrt{10}\cos(t - \arctan(\frac{1}{3}))$
$C_1 = 1$
$v_{g_2}(t) = \cos(t + \pi)$
$L_1 = 1$
$R_2 = 1$
$L_2 = 1$

Fasori

$\mathbf{V_{g_2}} = -1$
$\mathbf{I_{g_1}} = 3 - j$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_2 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{R_1} + Y_a)\mathbf{E_1} & -Y_a\mathbf{E_2} & -\frac{1}{R_1}\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ -Y_a\mathbf{E_1} & +(j\omega C_1 + \frac{1}{j\omega L_1} + Y_a)\mathbf{E_2} & -j\omega C_1\mathbf{E_3} & = & 0 \\ -\frac{1}{R_1}\mathbf{E_1} & -j\omega C_1\mathbf{E_2} & +(\frac{1}{R_1} + j\omega C_1)\mathbf{E_3} & = & -\mathbf{I_{g_1}} \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{3}{2}-\frac{1}{2}j)\mathbf{E_1} & +(-\frac{1}{2}+\frac{1}{2}j)\mathbf{E_2} & -\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ (-\frac{1}{2}+\frac{1}{2}j)\mathbf{E_1} & +(\frac{1}{2}-\frac{1}{2}j)\mathbf{E_2} & -j\mathbf{E_3} & = & 0 \\ -\mathbf{E_1} & -j\mathbf{E_2} & +(1+j)\mathbf{E_3} & = & -3+j \\ -\mathbf{E_1} & & & = & -1 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} & = & 1 \\ \mathbf{E_2} & = & -j \\ \mathbf{E_3} & = & j \\ \mathbf{I_{x_2}} & = & -2+j \end{cases}$$

Bilancio di potenza
Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = -\mathbf{E_3} = -j & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{1}{2} - \frac{3}{2}j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_1}} = -2+j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 1 + \frac{1}{2}j \end{array}$$

$P_{c_{tot}} = \frac{3}{2} - j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{R_1} = 1-j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{Z_a} = -1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{3}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{E_2} - \mathbf{E_3} = -2j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -2 \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_2}}{j\omega L_1} = -1 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{L_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{Z_a} = -1 & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -1 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{R_1} = 1 - j;$	$I_{R_1} = -1 + j;$	$Pa_{R_1} = 1$
$V_{g_1} = -j;$	$I_{g_1} = 3 - j;$	$Pc_{I_{g1}} = \frac{1}{2} - \frac{3}{2}j$
$V_{C_1} = -2j;$	$I_{C_1} = -2;$	$Q_{C_1} = -2$
$V_{g_2} = -1;$	$I_{g_2} = -2 + j;$	$Pc_{V_{g2}} = 1 + \frac{1}{2}j$
$V_{L_1} = -j;$	$I_{L_1} = 1;$	$Q_{L_1} = \frac{1}{2}$
$V_{R_2} + V_{L_2} = -1 - j;$	$I_{R_2} = I_{L_2} = 1;$	$Pa_{R_2} = \frac{1}{2}$
$Q_{L_2} = \frac{1}{2}$		