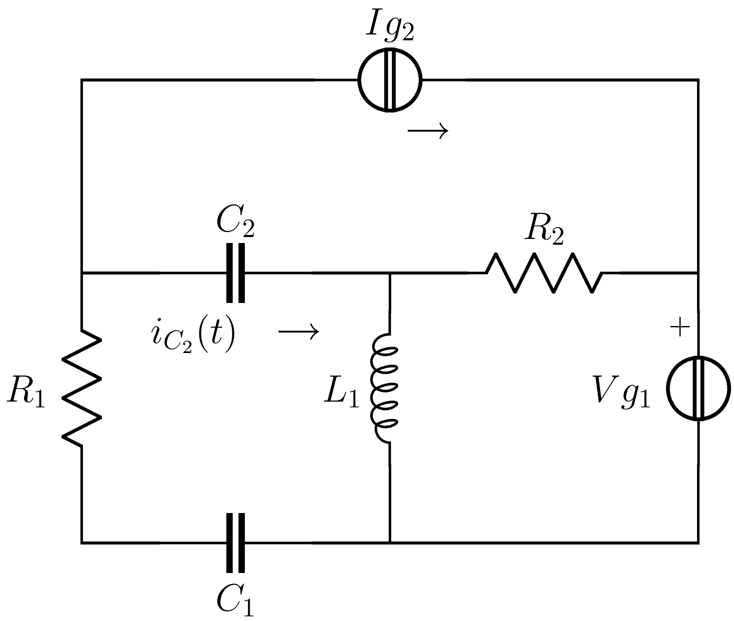
Esercizio b1 risolto

Risolvere il circuito in figura



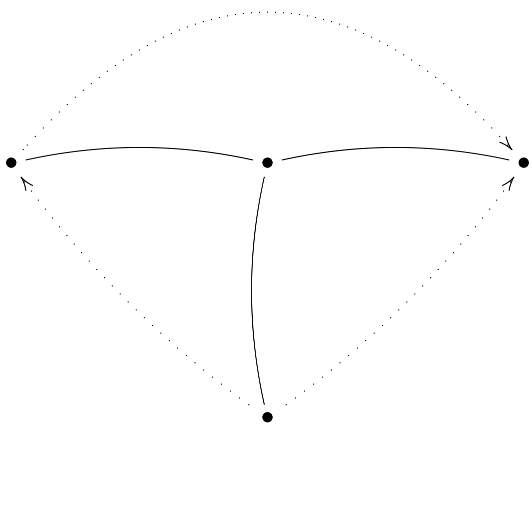
$R_1 = 1$
$C_1 = 1$
$L_1 = 2$
$\mathbf{V_{g_1}} = -2 - 6j$
$C_2 = 1$
$R_2 = 2$
$\mathbf{I_{g_2}} = 1 - j$
$\omega = 1$

Semplificazioni serie/parallelo

$Z_a = R_1 + \frac{1}{j\omega C_1} = 1 - j$
$Y_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (Z_a + j\omega L_1 + \frac{1}{j\omega C_2})\mathbf{I_1} & +j\omega L_1 \mathbf{I_2} & -\frac{1}{j\omega C_2}\mathbf{I_3} & = & 0 \\ j\omega L_1 \mathbf{I_1} & +(j\omega L_1 + R_2)\mathbf{I_2} & +R_2\mathbf{I_3} & = & \mathbf{V_{g_1}} \\ -\frac{1}{j\omega C_2}\mathbf{I_1} & +R_2\mathbf{I_2} & +(\frac{1}{j\omega C_2} + R_2)\mathbf{I_3} & = & \mathbf{V_{x_2}} \\ & & \mathbf{I_3} & = & \mathbf{I_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} \mathbf{I_1} & +2j\mathbf{I_2} & +j\mathbf{I_3} & = & 0 \\ 2j\mathbf{I_1} & +(2+2j)\mathbf{I_2} & +2\mathbf{I_3} & = & -2-6j \\ j\mathbf{I_1} & +2\mathbf{I_2} & +(2-j)\mathbf{I_3} & = & \mathbf{V_{x_2}} \\ & & \mathbf{I_3} & = & 1-j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & -1+j \\ \mathbf{I_2} & = & -1 \\ \mathbf{I_3} & = & 1-j \\ \mathbf{V_{x_2}} & = & -2-4j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_2} = -1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = 1+3j \\ \mathbf{V_{I_{g2}}} = \mathbf{V_{x_g1}} = -2-4j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = 1-3j \end{array}$$

$P_{c_{tot}} = 2$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_1} = -1+j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = -\mathbf{I_2} - \mathbf{I_3} = j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 1 \end{array}$$

$P_{a_{tot}} = 2 = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll}
\mathbf{I_{L_1}} = -\mathbf{I_1} - \mathbf{I_2} = 2 - j & Q_{L_1} = \frac{1}{2}\omega L_1 |\mathbf{I_{L_1}}|^2 = 5 \\
\mathbf{V_{C_2}} = (\mathbf{I_1} - \mathbf{I_3})\frac{1}{j\omega C_2} = 2 + 2j & Q_{C_2} = -\frac{1}{2}\omega C_2 |\mathbf{V_{C_2}}|^2 = -4 \\
\mathbf{V_{C_1}} = \frac{\mathbf{I_1}}{j\omega C_1} = 1 + j & Q_{C_1} = -\frac{1}{2}\omega C_1 |\mathbf{V_{C_1}}|^2 = -1
\end{array}$$

$Q_{tot} = 0 = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{C_2}} = \mathbf{I_1} - \mathbf{I_3} = -2 + 2j$$

$$i_{C_2}(t) = 2\sqrt{2}\cos{(t + \frac{3\pi}{4})}$$

Soluzioni:

$$\begin{array}{lll}
V_{R_1} + V_{C_1} = -2j; & I_{R_1} = I_{C_1} = -1 + j; & Pa_{R_1} = 1 \\
Q_{C_1} = -1 & & \\
V_{L_1} = -2 - 4j; & I_{L_1} = 2 - j; & Q_{L_1} = 5 \\
V_{g_1} = -2 - 6j; & I_{g_1} = -1; & Pc_{V_{g1}} = 1 + 3j \\
V_{C_2} = -2 - 2j; & I_{C_2} = -2 + 2j; & Q_{C_2} = -4 \\
V_{R_2} = -2j; & I_{R_2} = j; & Pa_{R_2} = 1 \\
V_{g_2} = -2 - 4j; & I_{g_2} = 1 - j; & Pc_{I_{g2}} = 1 - 3j
\end{array}$$