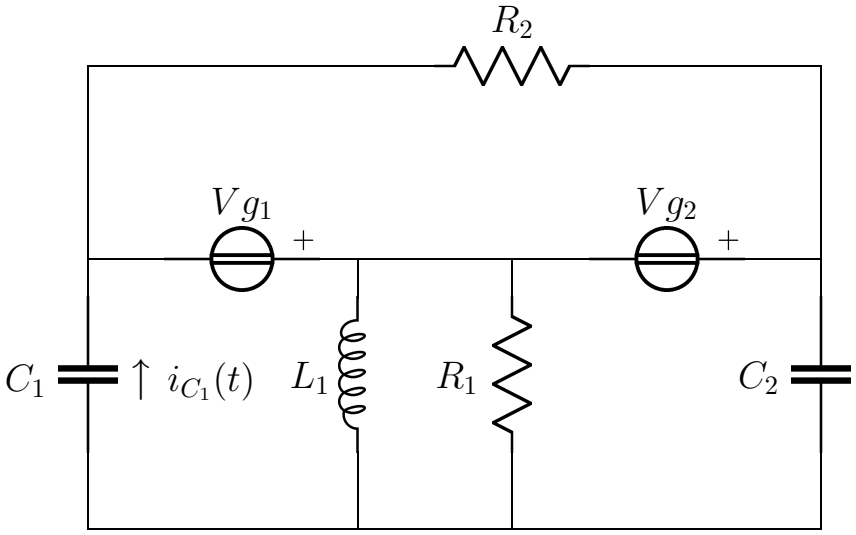
Esercizio C2 risolto

Risolvere il circuito in figura



$C_1 = 2$
$R_1 = 1$
$L_1 = \frac{1}{16}$
$C_2 = \frac{1}{2}$
$v_{g_1}(t) = \cos(2t)$
$v_{g_2}(t) = \sqrt{2}\cos(2t - \frac{\pi}{4})$
$R_2 = \frac{1}{2}$

Fasori

$\mathbf{V_{g_2}} = 1 - j$
$\mathbf{V_{g_1}} = 1$

Semplificazioni serie/parallelo

$Y_a = \frac{1}{R_1} + \frac{1}{j\omega L_1} = 1 - 8j$
$Z_a = \frac{1}{65} + \frac{8}{65}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (j\omega C_1 + \frac{1}{R_2})\mathbf{E_1} & -\frac{1}{R_2}\mathbf{E_2} & -j\omega C_1\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ -\frac{1}{R_2}\mathbf{E_1} & +(j\omega C_2 + \frac{1}{R_2})\mathbf{E_2} & -j\omega C_2\mathbf{E_3} & = & \mathbf{I_{x_2}} \\ -j\omega C_1\mathbf{E_1} & -j\omega C_2\mathbf{E_2} & +(j\omega C_1 + Y_a + j\omega C_2)\mathbf{E_3} & = & 0 \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_1}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (2+4j)\mathbf{E_1} & -2\mathbf{E_2} & -4j\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ -2\mathbf{E_1} & +(2+j)\mathbf{E_2} & -j\mathbf{E_3} & = & \mathbf{I_{x_2}} \\ -4j\mathbf{E_1} & -j\mathbf{E_2} & +(1-3j)\mathbf{E_3} & = & 0 \\ -\mathbf{E_1} & & & = & 1 \\ & \mathbf{E_2} & & = & 1-j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} & = & -1 \\ \mathbf{E_2} & = & 1-j \\ \mathbf{E_3} & = & 1 \\ \mathbf{I_{x_1}} & = & 4+6j \\ \mathbf{I_{x_2}} & = & 5-2j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_2}} = 4+6j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = 2-3j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_2}} = 5-2j & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I}^*_{\mathbf{V_{g2}}} = \frac{7}{2}-\frac{3}{2}j \end{array}$$

$P_{c_{tot}} = \frac{11}{2} - \frac{9}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{-\mathbf{E_3}}{R_1} = -1 & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{1}{2} \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{R_2} = 4-2j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 5 \end{array}$$

$P_{a_{tot}} = \frac{11}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{E_3} - \mathbf{E_1} = 2 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -8 \\ \mathbf{V_{C_2}} = \mathbf{E_2} - \mathbf{E_3} = -j & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -\frac{1}{2} \\ \mathbf{I_{L_1}} = \frac{-\mathbf{E_3}}{j\omega L_1} = 8j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 4 \end{array}$$

$Q_{tot} = -\frac{9}{2} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{C_1}} = (\mathbf{E_3} - \mathbf{E_1})j\omega C_1 = 8j$$

$$i_{C_1}(t) = 8\cos(2t + \frac{\pi}{2})$$

Soluzioni:

$V_{C_1} = -2;$	$I_{C_1} = 8j;$	$Q_{C_1} = -8$
$V_{R_1} = V_{L_1} = -1;$	$I_{R_1} + I_{L_1} = 1 - 8j;$	$Pa_{R_1} = \frac{1}{2}$
$Q_{L_1} = 4$		
$V_{C_2} = -j;$	$I_{C_2} = -1;$	$Q_{C_2} = -\frac{1}{2}$
$V_{g_1} = 1;$	$I_{g_1} = 4 + 6j;$	$Pc_{V_{g1}} = 2 - 3j$
$V_{g_2} = 1 - j;$	$I_{g_2} = 5 - 2j;$	$Pc_{V_{g2}} = \frac{7}{2} - \frac{3}{2}j$
$V_{R_2} = 2 - j;$	$I_{R_2} = -4 + 2j;$	$Pa_{R_2} = 5$