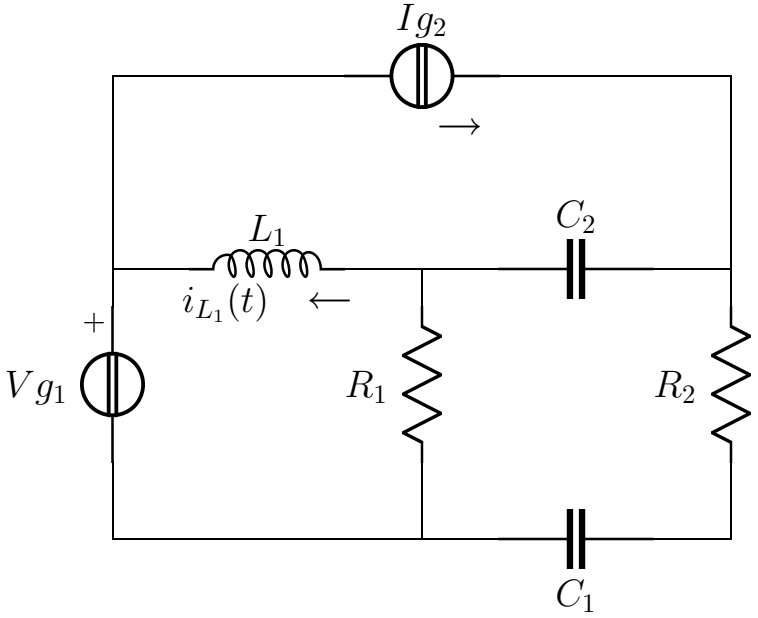
Esercizio C1 risolto

Risolvere il circuito in figura



$v_{g_1}(t) = 10\cos(2t + \arctan(\frac{3}{4}))$
$R_1 = 2$
$R_2 = \frac{1}{2}$
$C_1 = \frac{1}{2}$
$L_1 = 2$
$C_2 = 1$
$i_{g_2}(t) = \sqrt{2}\cos(2t + \frac{3\pi}{4})$

Fasori

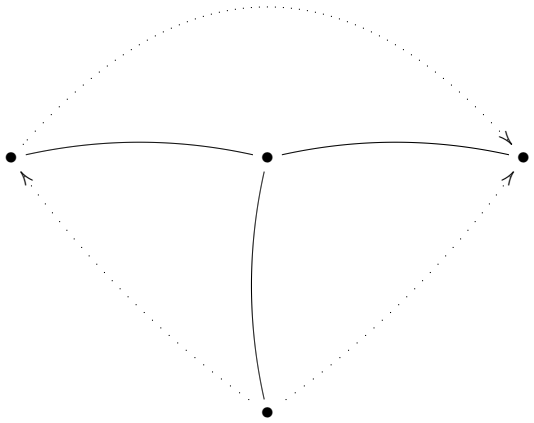
$\mathbf{I_{g_2}} = -1 + j$
$\mathbf{V_{g_1}} = 8 + 6j$

Semplificazioni serie/parallelo

$Z_a = R_2 + \frac{1}{j\omega C_1} = \frac{1}{2} - j$
$Y_a = \frac{2}{5} + \frac{4}{5}j$

Risoluzione dell'esercizio con il metodo delle maglie

Albero e coalbero:



Sistema

$$\begin{cases} (R_1 + j\omega L_1)\mathbf{I_1} & +R_1\mathbf{I_2} & -j\omega L_1\mathbf{I_3} & = & \mathbf{V_{g_1}} \\ R_1\mathbf{I_1} & +(R_1 + Z_a + \frac{1}{j\omega C_2})\mathbf{I_2} & +\frac{1}{j\omega C_2}\mathbf{I_3} & = & 0 \\ -j\omega L_1\mathbf{I_1} & +\frac{1}{j\omega C_2}\mathbf{I_2} & +(j\omega L_1 + \frac{1}{j\omega C_2})\mathbf{I_3} & = & \mathbf{V_{x_2}} \\ & & \mathbf{I_3} & = & \mathbf{I_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (2+4j)\mathbf{I_1} & +2\mathbf{I_2} & -4j\mathbf{I_3} & = & 8+6j \\ 2\mathbf{I_1} & +(\frac{5}{2} - \frac{3}{2}j)\mathbf{I_2} & -\frac{1}{2}j\mathbf{I_3} & = & 0 \\ -4j\mathbf{I_1} & -\frac{1}{2}j\mathbf{I_2} & +\frac{7}{2}j\mathbf{I_3} & = & \mathbf{V_{x_2}} \\ & & \mathbf{I_3} & = & -1+j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} & = & 1-j \\ \mathbf{I_2} & = & -1 \\ \mathbf{I_3} & = & -1+j \\ \mathbf{V_{x_2}} & = & -\frac{15}{2} - 7j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_1} = 1-j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = 1+7j \\ \mathbf{V_{I_{g2}}} = \mathbf{V_{x_2}} = -\frac{15}{2} - 7j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = \frac{1}{4} + \frac{29}{4}j \end{array}$$

$P_{c_{tot}} = \frac{5}{4} + \frac{57}{4}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = -\mathbf{I_1} - \mathbf{I_2} = j & P_{a_{R_1}} = \frac{1}{2} R_1 |\mathbf{I_{R_1}}|^2 = 1 \\ \mathbf{I_{R_2}} = \mathbf{I_2} = -1 & P_{a_{R_2}} = \frac{1}{2} R_2 |\mathbf{I_{R_2}}|^2 = \frac{1}{4} \end{array}$$

$P_{a_{tot}} = \frac{5}{4} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_1}} = -\mathbf{I_1} + \mathbf{I_3} = -2 + 2j & Q_{L_1} = \frac{1}{2} \omega L_1 |\mathbf{I_{L_1}}|^2 = 16 \\ \mathbf{V_{C_2}} = (-\mathbf{I_2} - \mathbf{I_3}) \frac{1}{j\omega C_2} = -\frac{1}{2} - j & Q_{C_2} = -\frac{1}{2} \omega C_2 |\mathbf{V_{C_2}}|^2 = -\frac{5}{4} \\ \mathbf{V_{C_1}} = \frac{\mathbf{I_2}}{j\omega C_1} = j & Q_{C_1} = -\frac{1}{2} \omega C_1 |\mathbf{V_{C_1}}|^2 = -\frac{1}{2} \end{array}$$

$Q_{tot} = \frac{57}{4} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{L_1}} = -\mathbf{I_1} + \mathbf{I_3} = -2 + 2j$$

$$i_{L_1}(t) = 2\sqrt{2} \cos\left(2t + \frac{3\pi}{4}\right)$$

Soluzioni:

$$\begin{array}{lll} V_{g_1} = 8 + 6j; & I_{g_1} = 1 - j; & Pc_{V_{g1}} = 1 + 7j \\ V_{R_1} = -2j; & I_{R_1} = j; & Pa_{R_1} = 1 \\ V_{R_2} + V_{C_1} = \frac{1}{2} - j; & I_{R_2} = I_{C_1} = -1; & Pa_{R_2} = \frac{1}{4} \\ Q_{C_1} = -\frac{1}{2} & & \\ V_{L_1} = -8 - 8j; & I_{L_1} = 2 - 2j; & Q_{L_1} = 16 \\ V_{C_2} = \frac{1}{2} + j; & I_{C_2} = 2 - j; & Q_{C_2} = -\frac{5}{4} \\ V_{g_2} = -\frac{15}{2} - 7j; & I_{g_2} = -1 + j; & Pc_{I_{g2}} = \frac{1}{4} + \frac{29}{4} j \end{array}$$