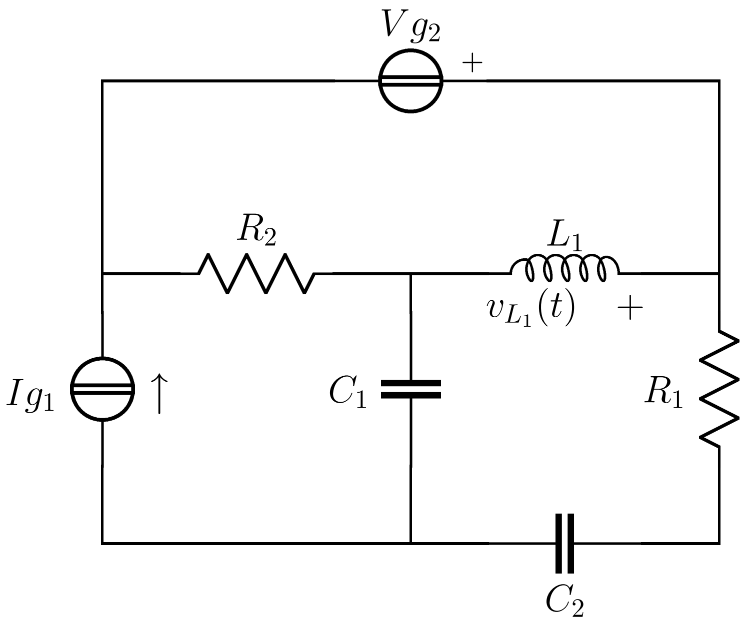
Esercizio salerno1

Risolvere il circuito in figura



$\mathbf{I_{g_1}} = -1$
$C_1 = 1$
$R_1 = \frac{3}{2}$
$C_2 = \frac{2}{5}$
$R_2 = 2$
$L_1 = 2$
$\mathbf{V_{g_2}} = 6 + 4j$
$\omega = 1$

Semplificazioni serie/parallelo

$Z_a = R_1 + \frac{1}{j\omega C_2} = \frac{3}{2} - \frac{5}{2}j$
$Y_a = \frac{3}{17} + \frac{5}{17}j$

Risoluzione dell'esercizio con il metodo delle maglie

Sistema

$$\begin{cases} (\frac{1}{j\omega C_1} + R_2)\mathbf{I_1} & +\frac{1}{j\omega C_1}\mathbf{I_2} & -R_2\mathbf{I_3} & = & \mathbf{V_{x_1}} \\ \frac{1}{j\omega C_1}\mathbf{I_1} & +(\frac{1}{j\omega C_1} + Z_a + j\omega L_1)\mathbf{I_2} & +j\omega L_1\mathbf{I_3} & = & 0 \\ -R_2\mathbf{I_1} & +j\omega L_1\mathbf{I_2} & +(R_2 + j\omega L_1)\mathbf{I_3} & = & \mathbf{V_{g_2}} \\ \mathbf{I_1} & & & = & \mathbf{I_{g_1}} \end{cases}$$

Sostituzione

$$\begin{cases} (2-j)\mathbf{I_1} & -j\mathbf{I_2} & -2\mathbf{I_3} & = & \mathbf{V_{x_1}} \\ -j\mathbf{I_1} & +(\frac{3}{2} - \frac{3}{2}j)\mathbf{I_2} & +2j\mathbf{I_3} & = & 0 \\ -2\mathbf{I_1} & +2j\mathbf{I_2} & +(2+2j)\mathbf{I_3} & = & 6+4j \\ \mathbf{I_1} & & & = & -1 \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{I_1} &= -1 \\ \mathbf{I_2} &= 1-j \\ \mathbf{I_3} &= 1 \\ \mathbf{V_{x_1}} &= 5 \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = \mathbf{V_{x_1}} = -5 & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{5}{2} \\ \mathbf{I_{V_{g2}}} = \mathbf{I_3} = 1 & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = 3+2j \end{array}$$

$P_{c_{tot}} = \frac{11}{2} + 2j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \mathbf{I_2} = 1-j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{3}{2} \\ \mathbf{I_{R_2}} = \mathbf{I_1} - \mathbf{I_3} = -2 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 4 \end{array}$$

$P_{a_{tot}} = \frac{11}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = (-\mathbf{I_1} - \mathbf{I_2})\frac{1}{j\omega C_1} = 1 & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{1}{2} \\ \mathbf{I_{L_1}} = \mathbf{I_2} + \mathbf{I_3} = 2-j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 5 \\ \mathbf{V_{C_2}} = \frac{\mathbf{I_2}}{j\omega C_2} = -\frac{5}{2} - \frac{5}{2}j & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -\frac{5}{2} \end{array}$$

$Q_{tot} = 2 = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{V_{L_1}} = (\mathbf{I_2} + \mathbf{I_3})j\omega L_1 = 2+4j$$

$$v_{L_1}(t) = 2\sqrt{5}\cos(t + \arctan(2))$$

Soluzioni:

$V_{g_1} = -5;$	$I_{g_1} = -1;$	$Pc_{I_{g1}} = \frac{5}{2}$
$V_{C_1} = -1;$	$I_{C_1} = j;$	$Q_{C_1} = -\frac{1}{2}$
$V_{R_1} + V_{C_2} = 1 + 4j;$	$I_{R_1} = I_{C_2} = 1 - j;$	$Pa_{R_1} = \frac{3}{2}$
$Q_{C_2} = -\frac{5}{2}$		
$V_{R_2} = 4;$	$I_{R_2} = -2;$	$Pa_{R_2} = 4$
$V_{L_1} = 2 + 4j;$	$I_{L_1} = -2 + j;$	$Q_{L_1} = 5$
$V_{g_2} = 6 + 4j;$	$I_{g_2} = 1;$	$Pc_{V_{g2}} = 3 + 2j$