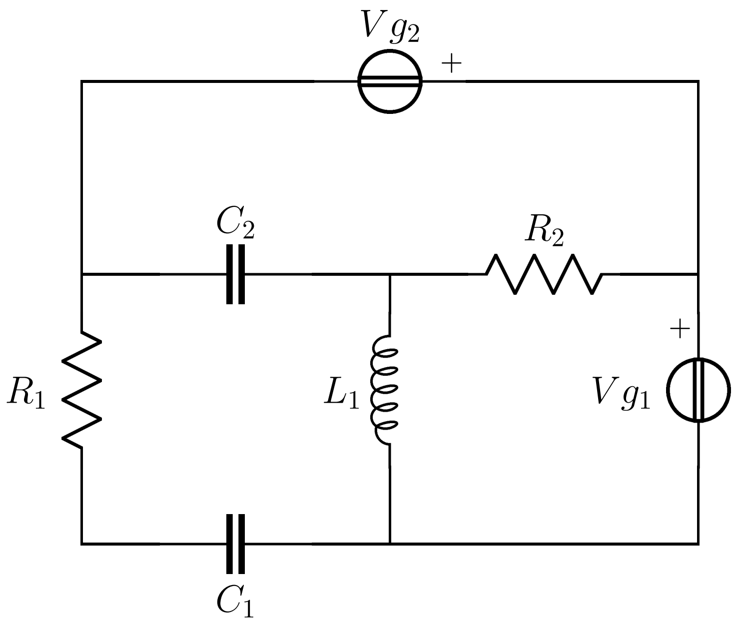
Esercizio c1 risolto 1

Risolvere il circuito in figura



$R_1 = \frac{3}{2}$
$C_1 = \frac{1}{11}$
$L_1 = 2$
$v_{g_1}(t) = 6\sqrt{2}\cos\left(2t + \frac{3\pi}{4}\right)$
$C_2 = \frac{1}{2}$
$R_2 = 2$
$v_{g_2}(t) = \sqrt{5}\cos\left(2t + \arctan(2)\right)$

Fasori

$\mathbf{V_{g_2}} = 1 + 2j$
$\mathbf{V_{g_1}} = -6 + 6j$

Semplificazioni serie/parallelo

$Z_a = R_1 + \frac{1}{j\omega C_1} = \frac{3}{2} - \frac{11}{2}j$
$Y_a = \frac{3}{65} + \frac{11}{65}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (Y_a + j\omega C_2)\mathbf{E_1} & -j\omega C_2\mathbf{E_2} & -Y_a\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ -j\omega C_2\mathbf{E_1} & +(\frac{1}{j\omega L_1} + j\omega C_2 + \frac{1}{R_2})\mathbf{E_2} & -\frac{1}{j\omega L_1}\mathbf{E_3} & = & 0 \\ -Y_a\mathbf{E_1} & -\frac{1}{j\omega L_1}\mathbf{E_2} & +(Y_a + \frac{1}{j\omega L_1})\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\ -\mathbf{E_1} & & & = & \mathbf{V_{g_2}} \end{cases}$$

Sostituzione

$$\begin{cases} (\frac{3}{65}+\frac{76}{65}j)\mathbf{E_1} & -j\mathbf{E_2} & +(-\frac{3}{65}-\frac{11}{65}j)\mathbf{E_3} & = & -\mathbf{I_{x_2}} \\ -j\mathbf{E_1} & +(\frac{1}{2}+\frac{3}{4}j)\mathbf{E_2} & +\frac{1}{4}j\mathbf{E_3} & = & 0 \\ (-\frac{3}{65}-\frac{11}{65}j)\mathbf{E_1} & +\frac{1}{4}j\mathbf{E_2} & +(\frac{3}{65}-\frac{8}{99}j)\mathbf{E_3} & = & -\mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & -6+6j \\ -\mathbf{E_1} & & & = & 1+2j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} & = & -1-2j \\ \mathbf{E_2} & = & -2-2j \\ \mathbf{E_3} & = & 6-6j \\ \mathbf{I_{x_1}} & = & j \\ \mathbf{I_{x_2}} & = & 1 \end{cases}$$

Bilancio di potenza
Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_g1}} = j & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I^*_{V_{g1}}} = 3+3j \\ \mathbf{I_{V_{g2}}} = \mathbf{I_{x_g1}} = 1 & P_{c_{V_{g2}}} = \frac{1}{2}\mathbf{V_{g2}}\mathbf{I^*_{V_{g2}}} = \frac{1}{2}+j \end{array}$$

$P_{c_{tot}} = \frac{7}{2}+4j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_1}-\mathbf{E_3}}{Z_a} = -1-1j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = \frac{3}{2} \\ \mathbf{I_{R_2}} = \frac{-\mathbf{E_2}}{R_2} = 1+j & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = 2 \end{array}$$

$P_{a_{tot}} = \frac{7}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_1}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{j\omega L_1} = 1+2j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = 10 \\ \mathbf{V_{C_2}} = \mathbf{E_2}-\mathbf{E_1} = -1 & Q_{C_2} = -\frac{1}{2}\omega C_2|\mathbf{V_{C_2}}|^2 = -\frac{1}{2} \\ \mathbf{V_{C_1}} = \frac{(\mathbf{E_1}-\mathbf{E_3})Y_a}{j\omega C_1} = -\frac{11}{2}+\frac{11}{2}j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -\frac{11}{2} \end{array}$$

$Q_{tot} = 4 = \Im m\{P_{c_{tot}}\}$

Soluzioni:

$V_{R_1} + V_{C_1} = -7 + 4j;$	$I_{R_1} = I_{C_1} = 1 + j;$	$Pa_{R_1} = \frac{3}{2}$
$Q_{C_1} = -\frac{11}{2}$		
$V_{L_1} = -8 + 4j;$	$I_{L_1} = -1 - 2j;$	$Q_{L_1} = 10$
$V_{g_1} = -6 + 6j;$	$I_{g_1} = j;$	$Pc_{V_{g1}} = 3 + 3j$
$V_{C_2} = -1;$	$I_{C_2} = j;$	$Q_{C_2} = -\frac{1}{2}$
$V_{R_2} = 2 + 2j;$	$I_{R_2} = -1 - j;$	$Pa_{R_2} = 2$
$V_{g_2} = 1 + 2j;$	$I_{g_2} = 1;$	$Pc_{V_{g2}} = \frac{1}{2} + j$