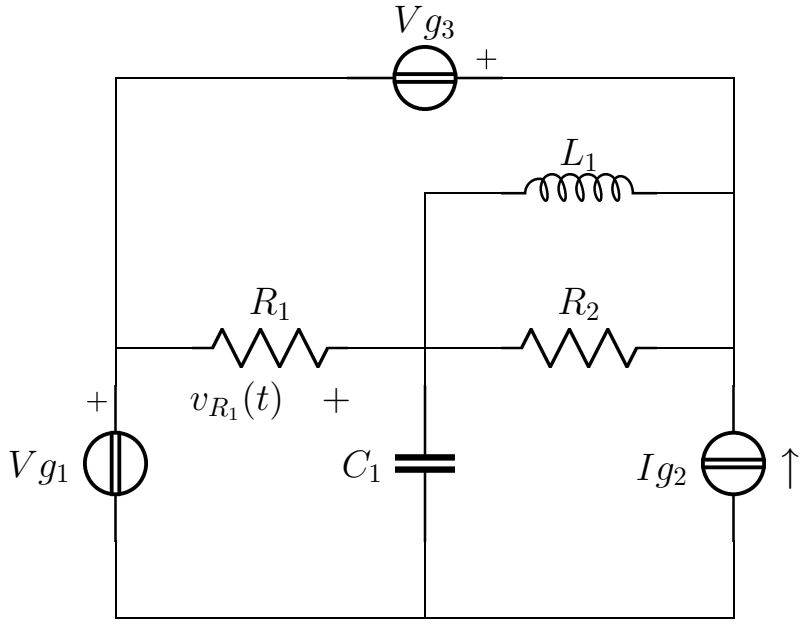
Esercizio A2 risolto

Risolvere il circuito in figura



$\mathbf{V_{g_1}} = -2$
$C_1 = 1$
$\mathbf{I_{g_2}} = -1$
$R_1 = 2$
$R_2 = 1$
$L_1 = 1$
$\mathbf{V_{g_3}} = 1 + 2j$
$\omega = 1$

Semplificazioni serie/parallelo
$Y_a = \frac{1}{R_2} + \frac{1}{j\omega L_1} = 1 - j$
$Z_a = \frac{1}{2} + \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi
Sistema

$$\begin{cases} (j\omega C_1 + \frac{1}{R_1} + Y_a)\mathbf{E_1} & -Y_a\mathbf{E_2} & -j\omega C_1\mathbf{E_3} & = & 0 \\ -Y_a\mathbf{E_1} & +Y_a\mathbf{E_2} & & = & \mathbf{I_{g_2}} + \mathbf{I_{x_3}} \\ -j\omega C_1\mathbf{E_1} & & +j\omega C_1\mathbf{E_3} & = & -\mathbf{I_{g_2}} - \mathbf{I_{x_1}} \\ & & -\mathbf{E_3} & = & \mathbf{V_{g_1}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_3}} \end{cases}$$

Sostituzione

$$\begin{cases} \frac{3}{2}\mathbf{E_1} + (-1+j)\mathbf{E_2} - j\mathbf{E_3} = 0 \\ (-1+j)\mathbf{E_1} + (1-j)\mathbf{E_2} = -1 + \mathbf{I_{x_3}} \\ -j\mathbf{E_1} + j\mathbf{E_3} = 1 - \mathbf{I_{x_1}} \\ -\mathbf{E_3} = -2 \\ \mathbf{E_2} = 1+2j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} = 2+2j \\ \mathbf{E_2} = 1+2j \\ \mathbf{E_3} = 2 \\ \mathbf{I_{x_1}} = -1 \\ \mathbf{I_{x_3}} = j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{I_{V_{g1}}} = \mathbf{I_{x_g 2}} = -1 & P_{c_{V_{g1}}} = \frac{1}{2}\mathbf{V_{g1}}\mathbf{I}^*_{\mathbf{V_{g1}}} = 1 \\ \mathbf{V_{I_{g2}}} = \mathbf{E_2} - \mathbf{E_3} = -1+2j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I}^*_{\mathbf{g2}} = \frac{1}{2} - j \\ \mathbf{I_{V_{g3}}} = \mathbf{I_{x_g 2}} = j & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I}^*_{\mathbf{V_{g3}}} = 1 - \frac{1}{2}j \end{array}$$

$P_{c_{tot}} = \frac{5}{2} - \frac{3}{2}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_1}}{R_1} = 1+j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 2 \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{R_2} = -1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{5}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{V_{C_1}} = \mathbf{E_1} - \mathbf{E_3} = 2j & Q_{C_1} = -\frac{1}{2}\omega C_1|\mathbf{V_{C_1}}|^2 = -2 \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_1}-\mathbf{E_2}}{j\omega L_1} = -j & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = -\frac{3}{2} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{V_{R_1}} = \mathbf{E_1} = 2+2j$$

$$v_{R_1}(t) = 2\sqrt{2}\cos\left(t + \frac{\pi}{4}\right)$$

$$\mathbf{I_{L_1}} = \frac{\mathbf{E_1} - \mathbf{E_2}}{j\omega L_1} = -j$$

$$i_{L_1}(t) = \cos\left(t - \frac{\pi}{2}\right)$$

Soluzioni:

$V_{g_1} = -2;$	$I_{g_1} = -1;$	$Pc_{V_{g1}} = 1$
$V_{C_1} = 2j;$	$I_{C_1} = 2;$	$Q_{C_1} = -2$
$V_{g_2} = -1 + 2j;$	$I_{g_2} = -1;$	$Pc_{I_{g2}} = \frac{1}{2} - j$
$V_{R_1} = 2 + 2j;$	$I_{R_1} = -1 - j;$	$Pa_{R_1} = 2$
$V_{R_2} = V_{L_1} = -1;$	$I_{R_2} + I_{L_1} = 1 - j;$	$Pa_{R_2} = \frac{1}{2}$
$Q_{L_1} = \frac{1}{2}$		
$V_{g_3} = 1 + 2j;$	$I_{g_3} = j;$	$Pc_{V_{g3}} = 1 - \frac{1}{2}j$