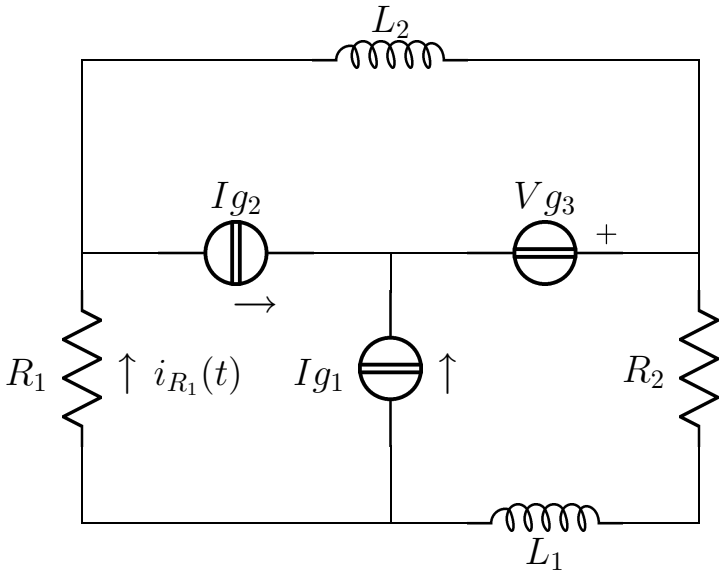
Esercizio B risolto

Risolvere il circuito in figura



$R_1 = \frac{1}{2}$
$\mathbf{I_{g_1}} = -5 + 2j$
$R_2 = 1$
$L_1 = \frac{1}{2}$
$\mathbf{I_{g_2}} = 3 - \frac{5}{2}j$
$\mathbf{V_{g_3}} = -j$
$L_2 = 1$
$\omega = 2$

Semplificazioni serie/parallelo

$Z_a = R_2 + j\omega L_1 = 1 + j$
$Y_a = \frac{1}{2} - \frac{1}{2}j$

Risoluzione dell'esercizio con il metodo dei nodi

Sistema

$$\begin{cases} (\frac{1}{R_1} + \frac{1}{j\omega L_2})\mathbf{E_1} & -\frac{1}{j\omega L_2}\mathbf{E_2} & -\frac{1}{R_1}\mathbf{E_3} & = & -\mathbf{I_{g_2}} \\ -\frac{1}{j\omega L_2}\mathbf{E_1} & +(Y_a + \frac{1}{j\omega L_2})\mathbf{E_2} & -Y_a\mathbf{E_3} & = & \mathbf{I_{x_3}} \\ -\frac{1}{R_1}\mathbf{E_1} & -Y_a\mathbf{E_2} & +(\frac{1}{R_1} + Y_a)\mathbf{E_3} & = & -\mathbf{I_{g_1}} \\ & \mathbf{E_2} & & = & \mathbf{V_{g_3}} \end{cases}$$

Sostituzione

$$\begin{cases} (2 - \frac{1}{2}j)\mathbf{E_1} & +\frac{1}{2}j\mathbf{E_2} & -2\mathbf{E_3} & = & -3 + \frac{5}{2}j \\ \frac{1}{2}j\mathbf{E_1} & +(\frac{1}{2} - j)\mathbf{E_2} & +(-\frac{1}{2} + \frac{1}{2}j)\mathbf{E_3} & = & \mathbf{I_{x_3}} \\ -2\mathbf{E_1} & +(-\frac{1}{2} + \frac{1}{2}j)\mathbf{E_2} & +(\frac{5}{2} - \frac{1}{2}j)\mathbf{E_3} & = & 5 - 2j \\ & \mathbf{E_2} & & = & -j \end{cases}$$

Soluzione

$$\begin{cases} \mathbf{E_1} = -1+j \\ \mathbf{E_2} = -j \\ \mathbf{E_3} = 1 \\ \mathbf{I_{x_3}} = -2-\frac{1}{2}j \end{cases}$$

Bilancio di potenza

Potenza complessa erogata dai generatori:

$$\begin{array}{ll} \mathbf{V_{I_{g1}}} = -\mathbf{E_3} = -1 & P_{c_{I_{g1}}} = \frac{1}{2}\mathbf{V_{I_{g1}}}\mathbf{I^*_{g1}} = \frac{5}{2}+j \\ \mathbf{V_{I_{g2}}} = -\mathbf{E_1} = 1-j & P_{c_{I_{g2}}} = \frac{1}{2}\mathbf{V_{I_{g2}}}\mathbf{I^*_{g2}} = \frac{11}{4}-\frac{1}{4}j \\ \mathbf{I_{V_{g3}}} = \mathbf{I_{x_2}} = -2-\frac{1}{2}j & P_{c_{V_{g3}}} = \frac{1}{2}\mathbf{V_{g3}}\mathbf{I^*_{V_{g3}}} = \frac{1}{4}+j \end{array}$$

$P_{c_{tot}} = \frac{11}{2}+\frac{7}{4}j$

Potenza attiva assorbita dai resistori:

$$\begin{array}{ll} \mathbf{I_{R_1}} = \frac{\mathbf{E_3}-\mathbf{E_1}}{R_1} = 4-2j & P_{a_{R_1}} = \frac{1}{2}R_1|\mathbf{I_{R_1}}|^2 = 5 \\ \mathbf{I_{R_2}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{Z_a} = -1 & P_{a_{R_2}} = \frac{1}{2}R_2|\mathbf{I_{R_2}}|^2 = \frac{1}{2} \end{array}$$

$P_{a_{tot}} = \frac{11}{2} = \Re e\{P_{c_{tot}}\}$

Potenza reattiva assorbita dai condensatori e induttori:

$$\begin{array}{ll} \mathbf{I_{L_2}} = \frac{\mathbf{E_2}-\mathbf{E_1}}{j\omega L_2} = -1-\frac{1}{2}j & Q_{L_2} = \frac{1}{2}\omega L_2|\mathbf{I_{L_2}}|^2 = \frac{5}{4} \\ \mathbf{I_{L_1}} = \frac{\mathbf{E_2}-\mathbf{E_3}}{Z_a} = -1 & Q_{L_1} = \frac{1}{2}\omega L_1|\mathbf{I_{L_1}}|^2 = \frac{1}{2} \end{array}$$

$Q_{tot} = \frac{7}{4} = \Im m\{P_{c_{tot}}\}$

Calcolo tensioni e correnti

$$\mathbf{I_{R_1}} = \frac{\mathbf{E_3}-\mathbf{E_1}}{R_1} = 4-2j$$

$$i_{R_1}(t) = 2\sqrt{5}\cos\left(2t-\arctan(\frac{1}{2})\right)$$

Soluzioni:

$V_{R_1} = -2 + j;$	$I_{R_1} = 4 - 2j;$	$Pa_{R_1} = 5$
$V_{g_1} = -1;$	$I_{g_1} = -5 + 2j;$	$Pc_{I_{g1}} = \frac{5}{2} + j$
$V_{R_2} + V_{L_1} = -1 - j;$	$I_{R_2} = I_{L_1} = 1;$	$Pa_{R_2} = \frac{1}{2}$
$Q_{L_1} = \frac{1}{2}$		
$V_{g_2} = 1 - j;$	$I_{g_2} = 3 - \frac{5}{2}j;$	$Pc_{I_{g2}} = \frac{11}{4} - \frac{1}{4}j$
$V_{g_3} = -j;$	$I_{g_3} = -2 - \frac{1}{2}j;$	$Pc_{V_{g3}} = \frac{1}{4} + j$
$V_{L_2} = 1 - 2j;$	$I_{L_2} = 1 + \frac{1}{2}j;$	$Q_{L_2} = \frac{5}{4}$